# Fuzzy Cognitive Mapping: Applications in Education

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Fostering conceptual and cognitive change in learners can be difficult. Students often come to a learning situation with robust, implicit understandings of the material under study. One explanation for the implicit nature of these understandings is a lack of metaknowledge about the knowledge to be acquired. Helping learners create metaknowledge may free paths to conceptual change. This paper proposes the use of fuzzy cognitive maps (FCMs) as a tool for creating metaknowledge and exploring hidden implications of a learner's understanding. Two specific educational applications of FCMs are explored in detail and recommendations are included for further investigations within educational contexts. © 2000 John Wiley & Sons, Inc.

"So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality." A. Einstein

## I. INTRODUCTION

Fuzzy cognitive mapping (FCM) is a tool for formalizing understandings of conceptual and causal relationships.[1] By combining conceptual mapping tools with fuzzy logic and other techniques originally developed for neural networks, FCMs allow for the representation and formalization of soft knowledge domains (e.g., politics, education). This paper explores FCM procedures and proposes two methodologies for developing FCMs in educational organization settings. Other potential applications in education are explored and directions for future research are included.

To apply FCMs in education requires a basic understanding of the theoretical foundation of cognitive mapping. This paper presents a brief review of that theoretical foundation, as well as some related research literature. Advantages

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of conceptual and semantic mapping are explored. Inherent weaknesses in current approaches are described. Most current mapping systems use a crisp approach (truth values of 1 or 0) to conceptual understanding and causal mapping. Crisp logic cannot accurately represent human understanding, especially in soft knowledge domains.

Fuzzy logic allows us to represent truth values on a continuous scale from 0 to 1, providing mathematical methods for representing concepts and causalities that are true to some degree (neither wholly true nor false). Consequently, the law of the excluded middle does not apply in fuzzy logic. Most human reasoning is fuzzy, with crisp distinctions as the special case of fuzzy logic. For example, when we say the water is very dirty, there is no hard line between *dirty* and *very dirty*. At some point the water is *both* dirty and very dirty. Fuzzy logic allows us to represent this idea mathematically and, thus, it becomes machine encodeable.

FCMs combine the strengths of cognitive maps with fuzzy logic. By representing human knowledge in a form more representative of natural human language than traditional concept mapping techniques, FCMs ease knowledge engineering and increase knowledge-source concurrence. FCMs can also be modeled on computers, thus allowing for dynamic modeling of cognitive systems.[2]

Two methods for facilitating the creation of FCMs are presented in this paper. Both are in the early stages of testing. The educational field testing includes trials of a method for group knowledge acquisition and for individual knowledge acquisition. In addition, a computer modeling system is presented for the development and analysis of FCMs.

## II. CONCEPTUAL CHANGE

Learners' naive conceptions have been well studied.[3,4] It is widely accepted within the domain of cognitive psychology that students come to school with some form of conceptualization of the natural world and their place in it. Frequently, however, these conceptions are not scientifically accurate. Instead, these conceptions represent a theory that is useful in everyday experience.[5] Naive theories are based on interaction with the everyday world. A child who repeatedly drops items on the floor is building an implicit theory of gravity. A child who tries to manipulate her parents into taking her for ice cream is building an implicit theory of human behavior.

Naive understandings display many of the characteristics of implicit knowledge. Implicit knowledge: (a) is characterized by specificity of transfer, (b) is associated with incidental learning conditions, (c) gives rise to a phenomenal sense of intuition, and (d) remains robust in the face of time, psychological disorder and secondary tasks.[6] Naive understandings meet many of the same criteria. They are learned incidentally, they give rise to a sense of "knowing," and remain robust in the face of time and schooling.[3] These naive understandings can be very difficult to diagnose and change.[4]

Much of the research on naive understandings has been in science education. The difficulty of changing conceptions of the natural world that have been formed over many years is well documented. Situations in which students are unable to process that which they are not expecting create opportunities for implicit learning. For example, when presented with science instruction, students often are looking to confirm what they "know" already, with the result both extremely selective attention and distortion of information provided during instruction.

Berry and Dienes[6] argue that implicit learning is in part characterized by a lack of metaknowledge. That is, we are unaware that we know what we know. The knowledge is unavailable in free recall tasks, but is generally available in performance, on forced choice tests and constrained answer tasks.[6] In order to make implicit knowledge available to the learner, some structured task must be available to elicit the knowledge from the learner.

Concept, or cognitive, mapping represents a possible tool for developing such a structured environment. The next section explores some of the methods currently used to create graphical formalisms of concepts.

## III. CONCEPT MAPPING

To promote conceptual change, some researchers have proposed using graphical representations of a specified conceptual domain. The current use of concept mapping within education has its roots in research conducted at Cornell that focused on conceptual changes in students over a 12 year period.[7] This research required a method to compare conceptions over time and between learners. The Cornell researchers developed a system of representing conceptual knowledge graphically: circles for concepts and arrows for the links between them.

A review of the literature[8–10] provides several definitions of concept maps, also known as cognitive maps. Two factors are common in these definitions: (a) all of the authors reviewed define a cognitive map as a graphical representation, and (b) most include some aspect of subjectivity. A graphical representation is fundamental to the idea of concept mapping. In one of the earliest references, Axelrod[8] developed a system for representing causal relationships in social science domains. The system represented concepts in sociology and political science as nodes in a directed graph. The nodes were connected by arrows that were assigned to represent positive or negative causal relationships. The other common factor in the definition of cognitive mapping is the subjectivity of the map. Irvine[9] describes concept mapping as the individual's diagrammatic interpretation of ideas. The definition from Park and Kim[10] concisely encapsulates many of these definitions.

> The cognitive map graphically represents interrelationships among a variety of factors. It is a representation of the perceptions and beliefs of a decision maker or expert about his/her own subjective world, rather than objective reality.[11]

Since the development of the graphical system at Cornell, there have been several studies conducted on the efficacy of using concept maps as teaching and learning tools. For example, Jegede, Alaiyemola, and Okebukola[12] report that students in Nigeria who used concept maps documented significantly higher mean scores on an achievement test for the subject matter studied. Other studies have concentrated on the use of concept mapping in teacher education. Novak[7] notes that most science teachers understand science to be a large body of information to be mastered, as opposed to a method for constructing new knowledge. Novak reports that concept mapping plays an important role in facilitating the change of science teachers' perception of science and the purpose of science education.

There are several factors that contribute to the power of cognitive mapping in learning. Johnson, Goldsmith, and Teague[13] describe two values categories of cognitive maps: the stimulus value and the structural advantage. The stimulus value is inherent in the graphical representation. Learners can easily see the global organization of the represented concepts. Graphical representations also allow for the organization of complex domains for learners and designers alike. The network structure of a concept map allows the simultaneous display of all the important relationships.[13] The structural advantage is relative to the assessment of pair-wise ratings. Johnson, Goldsmith, and Teague[13] report that assessing network representations of student understanding resulted in more valid measures than assessing pair-wise comparisons of concepts.

Thagard[5] proposes a system of conceptual change based on his historical research of scientific revolutions. The system delineates five levels of conceptual change, ranging from the simple to the complex:

(1) *Addition* of concepts.
(2) *Deletion* of concepts.
(3) *Simple reorganization* of concepts in the kind-hierarchy or part-hierarchy which results in new kind-relations and part-relations.
(4) *Revisionary reorganization* of concepts in the hierarchies, in which old kind-relations or part-relations are replaced by different ones.
(5) *Hierarchy reinterpretation*, in which the nature of the kind-relation or part-relation that constitutes a hierarchy changes.

Kind-relations define members of a concept (i.e., a whale is a kind of mammal), while part-relations define the characteristics of a member (i.e., whales have flippers). Concept maps make visible the potential for conceptual change within a learner. Thagard,[5] himself, makes good use of concept maps to demonstrate the conceptual change undergone by the scientists he studied.

Pressley and McCormick's[14] review of the literature on multiple representations in science revealed a common process for developing concept maps. This process is briefly outlined:

(1) *Key words and phrases are identified from the reading*.
(2) *Key concepts are ordered from the most general to the most specific*.
(3) *The concepts are then clustered using two criteria*. Concepts that interrelate are grouped; concepts are classified with respect to their level of abstraction (i.e., general concepts to specific ones). All of the concepts are then arranged loosely in a two-dimensional array with abstractness defining one dimension and main ideas defining the second dimension.

(4) *Related concepts are then linked with lines, which are labeled to specify the relationship between concepts.*

Thus, a map of the domain is created, with key ideas linked by nodes that describe the type of relationship between them (see Fig. 1). The map, however, is static. It represents the crystallized, declarative knowledge of a domain. The map can be used to understand some forms of the conceptual relationships. Dynamic, causal relationships, however, are beyond the scope of the methodology described by Pressley and McCormick.[14] Cognitive maps represent formal, bivalent (true or false) logical relationships. This is acceptable in "hard" knowledge domains like physics or mathematics where the nature of the knowledge in the domain is usually binary. In most domains, however, the knowledge base is uncertain, or fuzzy. Social studies (e.g., politics, international relations), management science, and the study of art or literature are all "soft" knowledge domains, where uncertainty and degrees of truth are more common. Thus, the true nature of these domains cannot be understood from bivalent, or true−false, concept maps.

Cognitive maps also represent a form of distributed intelligence. They are artifacts constructed to off-load complex tasks, structure activity, save mental work, and avoid error.[15] By creating a graphical representation of a domain, cognitive maps save the user from having to hold the representation in working memory, thus freeing cognitive resources for interpretation and analysis of the content.

In the remainder of this paper, we explore the extension of these concept maps as fuzzy cognitive maps (FCMs), a tool developed by Kosko,[2] for the representation of dynamic causal knowledge within soft knowledge domains.

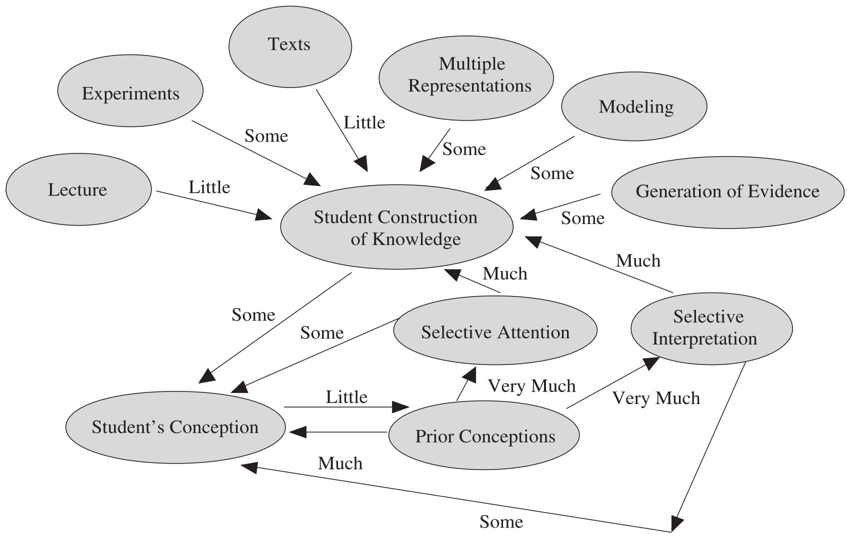


**Figure 1.** Fuzzy cognitive map. (Author interpretation of Pressley and McCormick[14]).

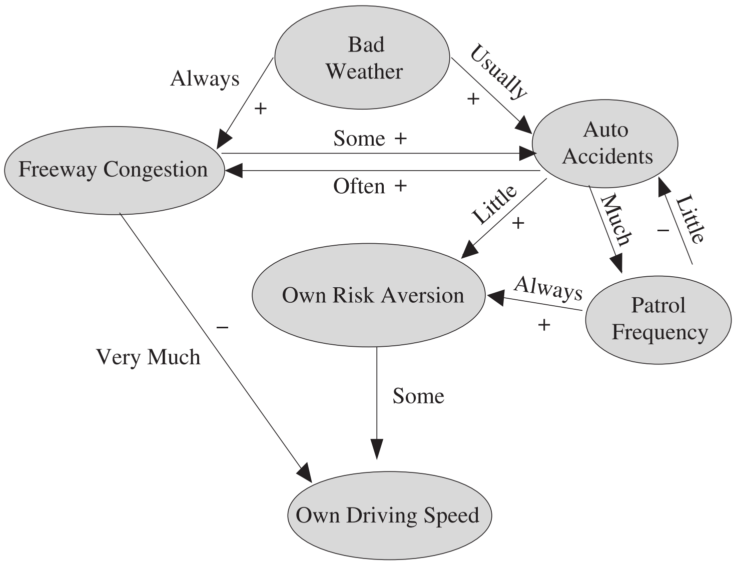


**Figure 2.** FCM for decisions about driving speed. (Adapted from Kosko[1]).

## IV. FUZZY COGNITIVE MAPS

Fuzzy cognitive maps (FCMs) and concept maps have similar applications in education. There are two important differences between traditional concept maps and FCMs: fuzzy logic and feedback. Uncertain or soft knowledge domains can be represented with fuzzy logic. Kosko[2] originally developed FCMs to represent concepts in military science, but his since gone on to demonstrate their usefulness in representing arguments in sociology and political science. Figure 2 is an adaptation of a map developed by Kosko[1] representing decisions about driving speed on a California highway. The dynamic nature of the FCM makes it a useful tool for discovering hidden relationships between concepts.[2] Variable input values can be entered into the nodes of the map in Figure 2 and analyzed via computer. The feedback systems within the map will gradually converge on a solution, or limit cycle. Interpreting the activation level of each node reveals relationships within the system. These features of FCMs will be explored in the next two sections.

Fuzzy logic is a system for representing uncertainty, or possibility. The formal extension of the original possibility theory created in the 1920s by Lukasiewicz[16] was developed by Zadeh.[17] A generalization of traditional, bivalent, Aristotelian logic, fuzzy logic creates a system for mathematically representing systems with natural linguistic variables (e.g., tall, little).

Traditional binary logic requires that a statement must be either true *or* false. An animal can be a cat *or* it is not a cat. The world of traditional logic is black or white. This is known as the law of the excluded middle: there is no option between 100% true and 100% false (see Fig. 3). The problem with a

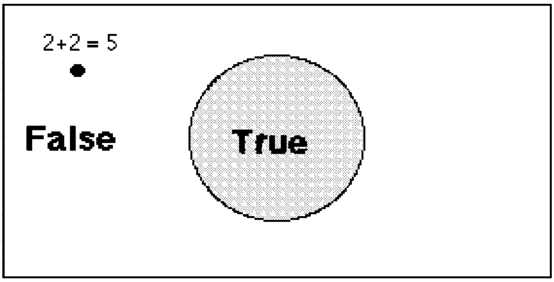


**Figure 3.** Binary logic representation.

binary system is that it does not allow for accurate representations of the way humans perceive and represent their world. Formal representations of human knowledge, like those described in the previous section, typically rely on binary logic.[16]

Fuzzy logic ignores the law of the excluded middle and allows for representations that are both true *and* false.[1] Terms like age, height, and intelligence are fuzzy variables; they have no exact definition and allow for degrees of membership. Fuzzy variables, or linguistic variables, have fuzzy values (e.g., young, short, bright).

A fuzzy set is a generalization of an ordinary set by allowing a degree of membership for each element. A membership degree is a real number on [0, 1]. In extreme cases if the degree of membership is 0 the element does not belong, and if 1 the element belongs 100% to the set.[18]

Thus, a person might be considered tall to a 0.9 degree if they were 6′5″ tall. Membership value is determined by a function that represents changing membership as a value changes (see Fig. 4).

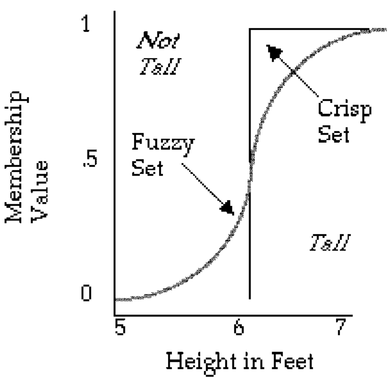


**Figure 4.** Fuzzy and crisp sets.

Kosko[2] claims it is possible to view thought as a fuzzy set, not as a language string. Given some space $X$ of primitives (i.e., sensibilia), a concept is some fuzzy subset of that space. This implies that for any given set of primitives, an infinitude of fuzzy subsets is possible. Perhaps the use of fuzzy logic allows us to create representations of human thought that are closer to the actual processes.

FCMs utilize fuzziness in several ways. First, the concepts themselves are fuzzy. In the example described in Figure 2, all of the nodes are fuzzy. Bad weather is a matter of degree, a light sprinkle to a hurricane. Freeway congestion can be heavy or light or anything in between. The causal edges themselves describe fuzzy functions. Auto accidents affect a driver's risk aversion to a small degree, while they affect patrol frequency to a large degree. Finally, the initial conditions that are inputs for the system are themselves fuzzy; they represent the initial degree of activation of a given node. The fuzziness in FCMs thus allows for the representation of ill-defined, complex concepts that are described using linguistic variables. The only price to pay is a fuzzy output.

### A. Causality and FCMs

Kosko[19] describes FCMs as fuzzy directional diagrams that illustrate feedback. Like traditional causal concept maps, FCMs have nodes that represent variable concepts. The links between the concepts are signed + or − to represent the nature of the relationship between nodes. Fuzzy logic allows the representation of fuzzy concepts and degree of causality. Feedback allows the user to explore the hidden properties of the map. By creating a formal representation of causality, FCMs can be used to create and explore models of dynamic events and search for causal explanation.

The most important difference between the concept maps which Pressley and McCormick[14] describe and FCMs is the temporal and causal nature of the FCM. FCMs express causality over time and allow for causality effects to fluctuate as input values change. Nonlinear feedback can only be modeled in a time-based system. FCMs are intended to model causality, not merely semantic relationships between concepts. The latter relationships are more appropriately represented with semantic networking tools like SemNet. By modeling causality over time, FCMs facilitate the exploration of the implications of complex conceptual models, as well as representing them with greater flexibility.

There is evidence to suggest that temporal mapping may improve learning. Lambiotte et al.[20] report that providing students with maps of procedures or processes, rather than semantic conceptual maps, improved learning; especially for lower ability students. FCMs are a tool for representing a dynamic process and modeling the process in real-time.

Investigating the results of an FCM model can also facilitate the discovery of causal explanations.[21] FCMs can be used as an artifact for graphically representing, and dynamically modeling, causal links. Computer modeling facilitates playing "what-if" games; allowing the learner to explore multiple contrastive questions. Unlike traditional concept maps, the dynamic properties of an FCM allow the user to play what-if games with the representation. By modeling the system on a computer, the user can rapidly explore a range of possibilities and potential system behaviors. Holding some parameters constant and varying other allows for the exploration of complex relationships that might not otherwise be investigated. Multiple perspectives from different experts may be explored by learners, offering them a greater range of perspectives than the current single perspective represented by computer-based instructional systems.

According to Miller,[21] for an explanation of any causal relationship to be appropriate it must meet two conditions: (a) the factors modeled must actually exist and be causally sufficient in typical cases under investigation, and (b) the pattern modeled must be the most accurate and reasonable place to stop searching for alternative answers. Finding such a pattern for complex relationships is a fundamentally impossible task. The search space of potential causal patterns for a given phenomena is nearly infinite, and is surely larger than most of us would care to explore. Thus, we rely on our current best guess, a causal pattern that is a local minima (or maxima, depending on your point of view). By distributing the cognitive load within the environment, FCMs expand the possible search space for reasonable patterns. FCMs balance the tendency of "best guess" by dynamically modeling the system, thus speeding the process of discerning which of the possible maps represent the given data and hold some degree of explanatory coherence.

Thagard's[5] levels of conceptual change correspond to the degree of reorganization of a given conceptual map. It is obvious when concepts are added or deleted. Simple reorganization creates new types of edges that can connect nodes. Revionary reorganization replaces one type of edge with another, and hierarchy reinterpretation changes the fundamental hierarchical relationship between nodes. Each of these processes can be supported with a visual representation. Additional research is needed to demonstrate the conditions under which this type of support is most effective.

Pressley and McCormick[14] emphasize the potential important cognitive outcomes if concept mapping does lead students to identify relationships they would otherwise miss and that identification allows the students to construct interpretations not obvious without mapping. The dynamic nature of the FCM makes it a useful tool for discovering hidden relationships between concepts.[22]

As discussed previously, FCMs represent an artifact for distributing cognition. The nonlinearity of a complex FCM is very difficult for us to manage consciously without additional support. The hidden relationships between nodes (relationships that are not immediately salient) are graphically represented in an FCM (e.g., the price of a postage stamp affects the cost of mass mailing which affects advertising budgets, but only after a time lag, and only if the economy is anemic…). "The resonant limit cycle… is a hidden pattern in the causal edges *E*. The hidden patterns in an expert's FCM presumably correspond to the expert's characteristic set of responses to what-if questions. As with an expert's answer, the resonant hidden pattern can be tested against the available evidence and the responsible FCM can be modified accordingly as needed."[24]

## B. Implementation

Representing and manipulating FCMs mathematically is not difficult. A given FCM with $C(n)$ concepts can be represented in an $N \times N$ matrix. Causality is represented by some nonlinear (usually sigmoidal) edge function $e(C_i, C_j)$, which describes the degree to which $C_i$ causes $C_j$. The edge function occurs over the bipolar interval $[-1, 1]$, as edges can be inhibitory or excitory. Using the notion of disconcepts ($\sim C$), the unit interval $[0, 1]$ can be retained.[19] Thus, what results is a matrix with causality between concepts represented by some real number between 1 and 0. A row, $i$, represents the causality between concept $i$ and all other concepts in the map. No concept is assumed to cause itself, thus the diagonal is zeroed. See Table I for an example of a simple FCM matrix.

Traditional cognitive mapping relies on a crisp valued edge function of $\{1, 0\}$ or $\{-1, 1\}$. Thus, when an expert creates a concept map, they sign the edges as either positive or negative. FCM representation create a weighted edge function over $[1, 0]$ or $[1, -1]$. Fuzziness, therefore, allows the developer to capture more fine grain information about the representation. It is possible to ask the expert to assign a real number weight to the edge, but this is difficult and usually unnecessary. Instead, an expert can use linguistic modifiers, which are then converted into fuzzy functions. Fuzzy inputs can be processed systematically via fuzzy causal algebra.[23]

Expert systems development typically involves only one expert due to difficulties with maintaining a tree structure and search limitations.[24] Kosko[19] has developed a mathematical method for combining the FCMs of multiple experts to represent a “field” view. Imagine combining FCMs from educational experts all over the world; creating a giant FCM for generating curriculum and research.

FCMs can be combined by representing them as matrices. Each expert’s map is represented as a matrix the size of the total number of unique concepts presented by all of the experts. The matrices are then added together, with the common links naturally achieving more weight. For example, Figure 5 represents an individual’s FCM generated during a pilot test of the group process described later. The corresponding matrix is presented in Table II. The addition of a second participant (see the matrix in Table III) is represented in Figure 6. In this example, both participants used the same concepts, as those were

**Table I.** A Simple FCM Matrix.

| To–From | Node 1 | Node 2 | Node 3 | Node 4 | Node 5 | Node 6 |
|---|---|---|---|---|---|---|
| Node 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| Node 2 | 0 | 0 | 1 | 0 | 0 | −1 |
| Node 3 | 0 | 1 | 0 | 1 | 1 | 0 |
| Node 4 | 0 | 0 | 1 | 0 | 1 | 0 |
| Node 5 | 0 | 0 | 0 | 0 | 0 | −1 |
| Node 6 | 0 | 0 | 0 | 0 | 0 | 0 |

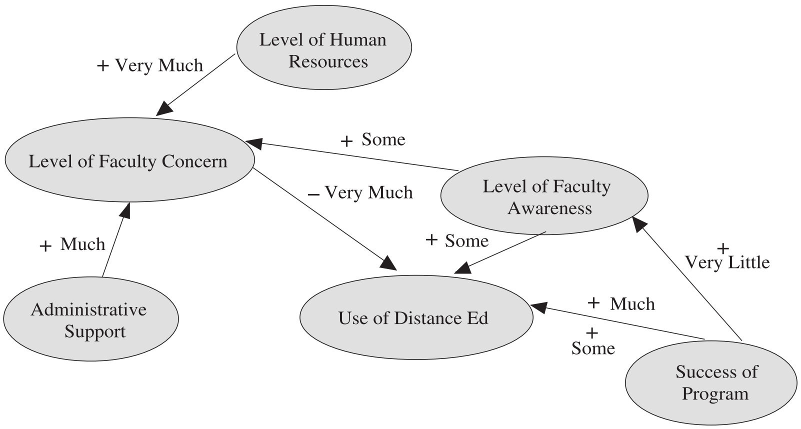


**Figure 5.** FCM of the use of distance education for participant #2.

**Table II.** FCM Matrix for Participant #2.

| To−From | Admin. Support | Faculty Concern | Human Resources | Faculty Awareness | Success of Program | Use of Distance Ed. |
|---|---|---|---|---|---|---|
| Admin. support | 0 | 4 | 0 | 0 | 0 | 0 |
| Level of faculty concern | 0 | 0 | 0 | 0 | 0 | −5 |
| Level of human resources | 0 | 5 | 0 | 0 | 0 | 0 |
| Level of faculty awareness | 0 | 3 | 0 | 0 | 0 | 3 |
| Success of program | 0 | 0 | 0 | 1 | 0 | 3 |
| Use of distance ed. | 0 | 0 | 0 | 0 | 0 | 0 |

**Table III.** Additive Matrix (Combined for Participants #2 and #3).

| To−From | Admin. Support | Faculty Concern | Human Resources | Faculty Awareness | Success of Program | Use of Distance Ed. |
|---|---|---|---|---|---|---|
| Admin. support | 0 | 4 | 4 | 0 | 0 | 0 |
| Level of faculty concern | 0 | 0 | 0 | 0 | 0 | −3 |
| Level of human resources | 0 | 0 | 0 | 0 | 5 | 0 |
| Level of faculty awareness | 0 | 3 | 0 | 0 | 0 | 3 |
| Success of program | 0 | 0 | 0 | 3 | 0 | 4 |
| Use of distance ed. | 0 | 0 | 0 | 0 | 3 | 0 |

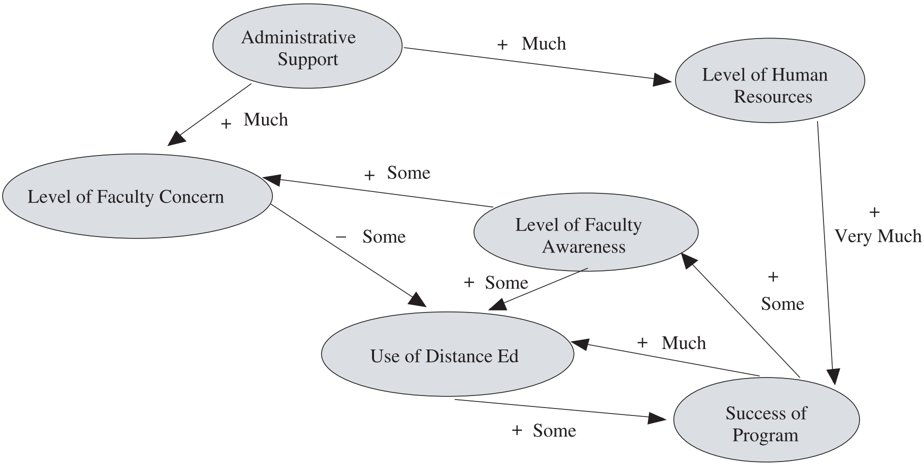


**Figure 6.** FCM of the use of distance education for participants #2 and #3.

generated in a group session. The resulting additive matrix is found in Table III. Notice that the links representing agreement have larger values, and the areas of disjunction or disagreement have smaller values. If there are $k$ experts, and only one expert includes a given edge, the maximum value for that edge is $1/k$. The results can then be normalized (averaged) over $[0, 1]$ or $[-1, 1]$.

In this system, if two experts perfectly disagree, they cancel each other out (i.e., if expert A says that edge $e_i$ is 1 and expert B says that $e_i$ is $-1$, the resulting edge equals 0). Large sample sizes tend to produce stable connection strengths.[22] Representations of the knowledge of multiple experts has long been a goal in expert systems development. By creating fuzzy knowledge structures, FCMs finally allow us to achieve this goal. Indeed, if large sample sizes produce stable connection strengths, then the more experts, the better.

An initial input activates the matrix. The initial state of the concepts is entered as a fuzzy vector. Inference proceeds by nonlinear spreading activation.[22] The initial activation is allowed to reverberate through the system until it converges on a limit cycle. See Table IV. The limit cycle may be a point solution, a cyclical attractor, or a chaotic strange attractor.[22] In other words, the output may be a steady state $(A, A, A, \ldots)$, a cycle $(A, B, C, A, B, C, \ldots)$, or a chaotic attractor $(A, C, B, D, B, A, D, C, \ldots)$. Figure 7 presents a graph of the output from one FCM. The transition to the limit cycle is evident as each line straightens and the system converges on the limit cycle.

Since FCMs are machine modeable and dynamic, they are also machine tunable. As the FCM is run through multiple what-if scenarios, adjustments to the nodes and edges can be made to gradually force the map to fit an expected output. Kosko[22] proposes a system of "adaptive inference through concomitant variation."[25] Instead of using simple Hebbian neural learning algorithms to tune the model, Kosko uses a differential Hebbian learning law that measures

**Table IV. Activated Matrix.**

| To–From | Admin. Support | Faculty Concern | Human Resources | Faculty Awareness | Success of Program | Use of Distance Ed. |
|---|---|---|---|---|---|---|
| Admin. support | 0 | 4 | 0 | 0 | 0 | 0 |
| Level of faculty concern | 0 | 0 | 0 | 0 | 0 | −5 |
| Level of human resources | 0 | 5 | 0 | 0 | 0 | 0 |
| Level of faculty awareness | 0 | 3 | 0 | 0 | 0 | 3 |
| Success of program | 0 | 0 | 0 | 1 | 0 | 3 |
| Use of distance ed. | 0 | 0 | 0 | 0 | 0 | 0 |

| To–From | Admin. Support | Faculty Concern | Human Resources | Faculty Awareness | Success of Program | Use of Distance Ed. |
|---|---|---|---|---|---|---|
| Admin. support | 0 | 4 | 4 | 0 | 0 | 0 |
| Level of faculty concern | 0 | 0 | 0 | 0 | 0 | −3 |
| Level of human resources | 0 | 0 | 0 | 0 | 5 | 0 |
| Level of faculty awareness | 0 | 3 | 0 | 0 | 0 | 3 |
| Success of program | 0 | 0 | 0 | 3 | 0 | 4 |
| Use of distance ed. | 0 | 0 | 0 | 0 | 3 | 0 |

| To–From | Admin. Support | Faculty Concern | Human Resources | Faculty Awareness | Success of Program | Use of Distance Ed. |
|---|---|---|---|---|---|---|
| Admin. support | 0 | 4 | 2 | 0 | 0 | 0 |
| Level of faculty concern | 0 | 0 | 0 | 0 | 0 | −4 |
| Level of human resources | 0 | 2.5 | 0 | 0 | 2.5 | 0 |
| Level of faculty awareness | 0 | 3 | 0 | 0 | 0 | 3 |
| Success of program | 0 | 0 | 0 | 2 | 0 | 3.5 |
| Use of distance ed. | 0 | 0 | 0 | 0 | 1.5 | 0 |

changes in the environmental parameters. A simple Hebbian law would be something like:

$$\tilde{X}_i = -X_i + \boldsymbol{+}_j\, C_j(X_j)e_{ji} + I_i$$

where $\tilde{X}_i$: is the activation level of some node $i$, $-X_i$: is the passive causal decay parameter, $C_j$: is a sigmoid function, $\boldsymbol{+}_j\, C_j(X_j)e_{ji}$: is the path-weighted internal feedback, $I_i$: is the external output.

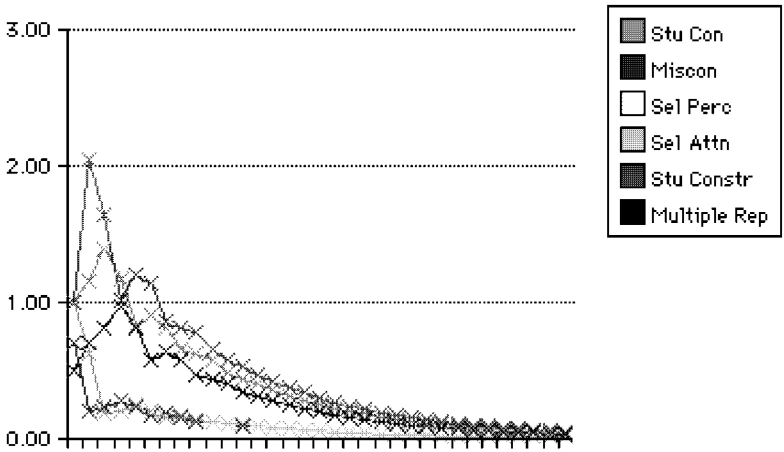


**Figure 7.** Graph of an activated matrix converging on a limit cycle.

A differential Hebbian learning law, on the other hand, is represented by:

$$e_{ji} = -e_{ji} + C_i{}'C_j{}'\,\tilde{}x_i\,\tilde{}x_j$$

where $e_{ji}$ = the edge function between concept $C_i$ and $C_j$

$$C_i' = dC_i/dx_j$$

Simulations show that while the simple Hebbian learning law (which correlates activations to output) produces "spurious causal conjectures,"[25] the differential Hebbian law causes $e_{ji}$ to converge on an exponentially weighted average of correlated change.

Using such techniques, FCMs can be gradually improved over time. Non-salient improvements that would probably have been missed by a human observer may be added to the map as it becomes more and more accurate.

## C. Advantages of FCMs

The previous sections have explored several differences between the concept mapping process described by Pressley and McCormick[14] and the FCM methodologies described in this paper. First, concepts in an FCM are not arrayed according to abstractness or centrality of the idea. The centrality of an idea can be naturally determined after the map has been completed. Centrality becomes a function of the number of links to and from a given node and the weight of those links. The abstractness of an idea can be interpreted as a function of its fuzziness. The more abstract an idea, the more fuzzy subsets it contains. Hierarchical conceptual relationships can be embedded within a FCM node. The node then becomes an embedded FCM within the larger framework. The resulting signal strength from the node is a function of the embedded processing. These features offer several advantages to FCMs over traditional mapping methods. FCMs have these specific advantageous characteristics:

(1) FCMs capture more information in the relationships between concepts.
(2) FCMs are dynamic.
(3) FCMs express hidden relationships.
(4) FCMs are combinable.
(5) FCMs are tunable.

## V. FCMs IN PRACTICE

Potential applications of FCMs are very broad. For the purposes of this paper, we will investigate two categories of potential applications. First, FCMs may be used in an organizational context to promote investigation by participants of their individual, deeply held assumptions, and as a tool for facilitating the adoption of new innovations. Second, FCMs have potential applications in intelligent tutoring systems. The reader should note that most of these applications are in the early stages of development and field testing. The discussions that follow represent potential, unproved applications.

### A. FCMs in an Organizational Context

Most of the authors' current work is focused on developing an organizational implementation of fuzzy cognitive mapping. The power of FCMs to reveal hidden patterns in complex conceptual maps can be exploited to promote institutional learning. Arie De Geuss, former planner for Shell Oil, defines institutional learning as changing shared mental models.[26] Developing shared representations of current and future mental models is a complex task. Strategic decision making environments are complex, unstructured, and not readily quantifiable.[27] Cognitive maps have been used as a decision making tool in international relations, administrative science, management science, and operations research.[10]

FCMs can be used to make the mental maps of management teams, and others, visible. When the implicitly held assumptions of the participants are laid bare, the process of exploring and changing mental models may be facilitated. Long-term change comes from building the models and participating in the process. Planning and decision making can also be facilitated as the models are tuned and adjusted over time. FCMs may not be the "magic bullet" for organizational learning, but the technique definitely shows promise as a tool for investigating organizational paradigms and promoting organizational learning.

The next section outlines a process recently field tested for group generation of FCMs. It was used to investigate faculty perceptions of the factors that affect the use of distance education at a medium-sized university. The workshop took place over 2 days and required roughly 8.5 h. At the end of the 2 days, individual FCMs were generated and a group consensus FCM was developed and analyzed to reveal factors which the faculty identified as important in the implementation of distance education.

## VI. DRAFT PROCESS FOR THE DEVELOPMENT OF GROUPS FCMs

### A. Introduction (30 min)

The session begins with a brief introduction to the methodology. The facilitator explains what we hope to achieve, the basic steps, and the rationale for this method.

*What we hope to achieve*: We hope this process achieves several results:

(1) An FCM of participant's thoughts about the use of distance education at this university.
(2) Participants will begin to articulate their concerns about distance education in an open forum.
(3) Open lines of communication among faculty about the process.

*Rationale*: We have chosen to use this process because we believe it will least bias participant responses. By allowing the participants to develop their own model of distance education usage at this university, we, the researchers, avoid biasing them with a preconceived model. Certainly, there are other ways we could achieve this without the FCM approach. The advantage of the FCM approach is that it allows us to explore the model generated and investigate hidden properties.

At this point, the goal is to address concerns and promote buy-in to the method, not to the use of distance education. The facilitator should be sure to communicate that the purpose is to gather honest information and belief statements. After a suitable period for questions and answers (thus the 30 min time frame), the facilitator should outline the basic ground rules for a brainstorming session. Remind participants that no kicking, scratching, or biting is allowed.

The goal of brainstorming is the free flow of ideas, thus: (a) say whatever comes to mind, no matter how silly it may seem at the time; (b) do not edit yourself or others. Voice your thoughts, we can edit and critique later; and (c) try to let all participants voice ideas.

### B. Brainstorming (45 min)

The brainstorming session is rather straightforward. The facilitator should encourage participation from everyone, and prevent editing or censoring. If the brainstorming energy seems to lag, the facilitator can either: (a) offer a general area of interest for participants to explore; (b) let the silence reign for a few moments; or (c) end the session if it appears the participants have put up everything they want to. Participant reactions–ideas are recorded on large sheets displayed around the room.

### C. Individual Clustering (15 min)

After the brainstorming session, the participants should be given a short break while idea cards are generated. The idea cards could have all of the ideas on them, or random subsets of the ideas. After the idea cards are ready, the participants should gather to be given the cards and a sheet of tacky paper. The facilitator should instruct them to create some preliminary clusters of ideas that seem alike. The facilitator should remind participants that the idea clusters are to *represent ideas that are conceptually similar*, *not ideas that affect each other*. A sign to this effect should be displayed at this time. The participants should be warned that they have only 15 min to complete this clustering and that they should not expect to finish in that time. This exercise is meant to give the participants a sense of the scope of the ideas as well as a chance to begin organizing ideas in their own minds. The final clusters will be generated by the group.

### D. Group Clustering (90 min)

Once the individual clusters are loosely fashioned, the large group should be reconvened. The facilitator should then explain that the group must come up with clusters of the ideas, preferably no more than 10. Again, remind the participants that they are looking for ideas that are conceptually similar, not ideas that impact each other.

The facilitator should begin by asking the group for some suggestions for starting clusters; ones developed individually. Each suggestion should be generally agreed to by the whole group. The facilitator will write each agreed upon cluster name on a large post-it sheet. Once most of the clusters have been agreed upon by the group, the facilitator should break the participants into groups of four or five and have them place labels (with the brainstormed ideas) under the appropriate cluster. Each group is given one set of labels. Each label must be placed in a cluster based on a consensus.

After the groups have finished clustering, the facilitator will look for items of agreement and disagreement among the groups. When there is any disagreement among groups about the placement of a brainstormed idea, each group will give a rationale for placing the idea(s) in question within a particular cluster. Conversation should continue until consensus is reached on all ideas by all groups. This activity helps participants define each cluster, thus ensuring that all are speaking "the same language."

The facilitator should take care to ensure maximum participation from the whole group. Participants who are reticent should be given explicit opportunities to speak. After the participants have finished the group clustering, allow participants to take a short break. (These breaks not only give the research–facilitation team an opportunity to process the most recent results, but also serve to encourage increased communication and clarification of ideas among participants.) After the break, each participant receives a map of the clusters generated by the large group. Participants are asked to take these home and think about how these clusters might be causally linked and to begin to place modifiers with a + or − directional link between clusters. They should be instructed to use the list of fuzzy modifiers supplied with the cluster maps that were generated prior to the workshop. A more advanced option would allow them to generate their own fuzzy modifiers.

## VII. DAY 2

### A. Second Thoughts (20 min)

The activities for the second day begin as the facilitator asks for second thoughts about the clusters that were generated the day before. Any suggestions for change should be discussed by the group and agreed upon by the group. It is not unusual for changes to occur after a period of individual reflection. The facilitator should also ask participants about their success in transitioning the group map into a causality map.

### B. Causality Mapping (90 min)

The next task is to generate an initial causality map for the whole group. A large map of the clusters should be created before the second thoughts activity. Encourage volunteers to discuss their own maps as starting points for the group. The large map should then be edited by the group until consensus is reached. The revised model should then be copied and distributed to the group.

Data gathering might end here. By this point of the pilot project, we had generated a fairly complex FCM of the faculty concerns (see Fig. 8) which allowed us to answer our research question. However, other goals outlined at the beginning of this method were left unfulfilled; specifically, opening lines of communication. To facilitate open communication and to give the faculty some sense of closure, the next two steps were important. The next steps also allow the researchers to expand the understanding of the potential impact of the model within the larger system. The time necessary for these steps is indeterminate. If successful, the communication channels will remain open beyond the close of the workshop.

### C. Implications

The researchers should have begun this second day with a rough matrix of the initial group cluster map in some computer-based format (e.g., spreadsheet, custom software). As the map progresses through variations (transitioning to a causality map), the researchers are quietly updating the computer-based matrix. When there is consensus around the final causality map, some initial system values are entered into the matrix and allowed to reverberate until a limit cycle

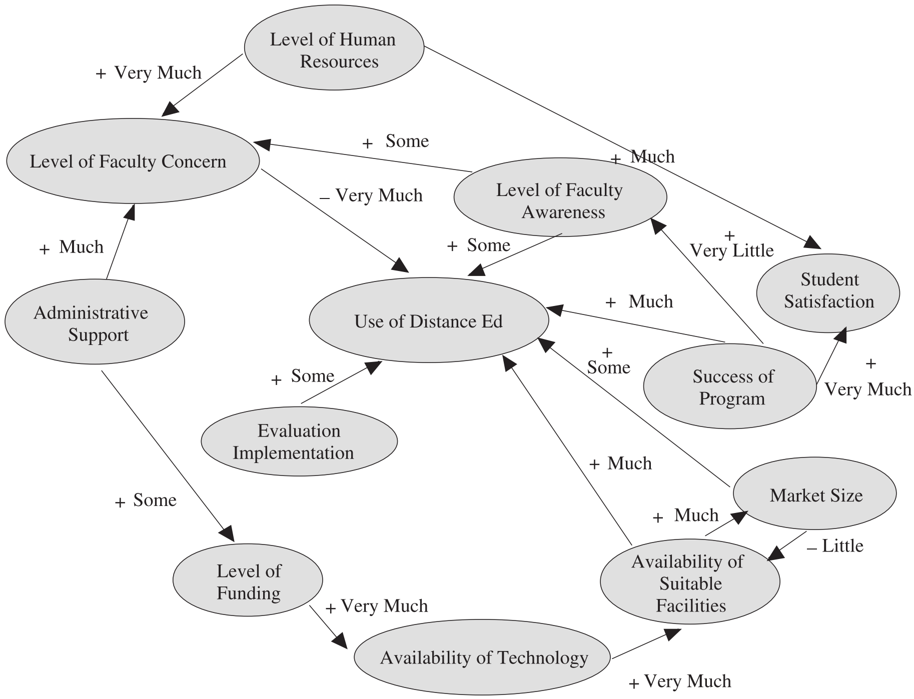


**Figure 8.** Group FCM of the use of distance education.

is reached. Participants have an opportunity to view the resulting model, discuss the impact, suggest alternative impacts, and generate other graphic scenarios for critique. Thus, participants are allowed to explore the hidden features of the model that they have collectively created.

Perhaps more valuable, though, is the participants' discussion of the ramification of the model. Questions to consider include:

(1) Which factors–ideas seem to be most important (e.g., have the most links, the strongest links)?
(2) Which factors–ideas can be controlled by individuals within the group, the institution, or by others outside the institution?
(3) What could be done to influence the factors–ideas that are under some control of the individual?

## D. Next Steps

Allow participants to discuss the course of best action. This discussion should correlate highly with the models and reflections generated with the computer-based matrix manipulations.

### *1. Results of the Field Test*

The workshop was held over a day and a half period late in the fall semester. The researchers invited 20 faculty and administrators to participate. The invitees represented a cross section of College of Education faculty, from novice teachers to more experienced. There are much discussion about the composition of the participant group, their experience, knowledge about distance education, and whether the sample should be random or representative. When the researchers realized that the primary goals of the process were qualitative rather than quantitative (e.g., fostering open communication), a representative sample was deemed more appropriate. Invitations were then extended, requesting voluntary participation. To support this effort, the Dean of the college also sent a letter endorsing the project and asking faculty to participate. Unfortunately, the workshop was scheduled during final exam week which created scheduling conflicts for many of the invited faculty.

On the first day of the workshop, four faculty participants arrived at a selected off-campus site to begin the process. Participants were faculty from four distinct departments and had very little background knowledge about distance education. These researchers facilitated the process and led the group through the process as previously described.

Over the course of the day and a half workshop, the participants generated a 12-node, interconnected FCM (see Fig. 8) of the factors they felt influenced the use of distance education technologies at the University. The brainstorming efforts took longer than anticipated (even with the small number of participants) due to the sheer number of items generated by participants. On the second day, reaching consensus on the weights and edges took much longer than scheduled. This part of the process had not been tested previously and will need to be adapted for future workshops. The discussions generated as part of the consensus building were valuable to the general understanding of the participants, but the amount of time required precluded engaging in the final steps of discussing and exploring the final group map.

During the field test, many possibilities for improving the process became apparent. Creating connections between the nodes took far too long, as did the consensus building. Other weaknesses were revealed in the balance between individual and group processes. We attempted to do too much in this first trial.

Analysis of the final group map yields some interesting observations. The most influential node on the use of distance education at the University, both in terms of weight and number of connections, was the node that represented levels of faculty comfort with the technology. The second most influential node was level of institutional support. These results can be interpreted as supporting existing theories about the adoption and diffusion of new technology. Faculty participants had a deep sense of the importance of the comfort node with relatively low concern for the hardware systems themselves.

An alternative interpretation is that the final group map reflects the general stage of adoption within the organization. The nodes that indicate that the faculty are most concerned with support from the institution and their own level of comfort indicate the organization is in a very early adoption phase; one characterized by potential adopters that are somewhat aware of the technology, but unsure about the potential impact on their professional lives.

A third potential interpretation is that faculty are naturally concerned with their own issues and would naturally make those most important to their use of the technology. They would naturally not be as concerned with issues that might be important to other populations (e.g., technical support staff, administration).

All participants reported that participation in the workshop had increased their awareness and understanding of the complexities of distance education at the University. They also praised the process for accommodating individual perspectives and allowing for nonadversarial problem solving on a potentially emotional topic.

### 2. *Applications in Intelligent Tutoring*

The promise of intelligent tutoring systems to create individualized instructional environments has spurred the creation of a plethora of demonstrations, research tools and prototypes.[28,29] Most systems to date have centered on hard knowledge domains; domains where the information is easily coded into the system. Anderson's LISP tutor[50] is a frequently cited example of an intelligent system. The advantage of creating systems in hard knowledge domains is the relative ease of creating an expert model and modeling the student representation. Typically the model takes the form of production rules[30]; a series of IF−THEN rules which encode the experts knowledge within the domain.

Typically, developing the expert cognitive model is the most time intensive task of developing an intelligent tutoring system. The task is analogous to developing an expert system.[30] FCMs present an alternative to rule-based expert systems.[19] Having an expert draw diagrams of their knowledge, rather than listing rules, is cognitively efficient and all maps share a common structure; facilitating knowledge combination between experts.[24] FCMs can also represent soft knowledge domains with the use of fuzzy concepts. Fuzziness allows the representation of hazy degrees of causality between hazy causal objects.[2]

The other expert system in an intelligent tutoring system is a system for modeling the learner's understanding. The intelligence of a system resides in its ability to create an accurate model of student understanding and "the ability to analyze learners' solution histories dynamically, using principles rather than preprogrammed responses to decide what to do next".[31] Again, this is commonly achieved with rule-based expert systems. Typically these rules are heuristics that apply generally over limited domains or probabilistic models of student achievement.[32] FCMs can represent a dynamic model of the student and student knowledge. In an analogous task, Lee, Kim, and Sakawa[33] developed a method of using FCMs for on-line fault diagnosis in a chemical processing plant. Using the temporal, dynamic aspects of FCMs, they generate a fault vector that is then matched against an appropriate FCM. The FCM can predict incipient faults and diagnose the cause of the fault vector. The authors found that FCMs were better tools than heuristic, branching models for representing the dynamic feedback loops in a complex chemical processing environment. While Lee, Kim, and Sakawa[33] made some critical assumptions in their work which are only applicable to a chemical plant, the general framework serves as a starting point for developing similar systems for cognitive diagnosis in an intelligent tutoring system.

The most advanced expert models are useless, however, without an instructional environment in which to embed them. Sugrue and Clark[34] discuss six categories of instructional methods that a fully supportive learning environment would need to support:

(1) Elaborate on the goal of the task and its' demands.
(2) Provide information related to the task.
(3) Provide practice tasks and contexts.
(4) Monitor trainee performance.
(5) Diagnose sources of error in performance.
(6) Adapt goal elaboration, information and practice tasks.

There are a number of potential applications of FCMs in a system that supports these instructional methods. The expert model might be used for diagnosis and monitoring in conjunction with the diagnostic models. FCMs can store rules for adapting the system to the learner and provide information related to the given practice tasks. Additional applications of FCMs include:

(1) *Elaborate on the goal of the task*—When FCMs are used to generate curricula, the goal driving the task becomes evident. A common complaint of students is that tasks frequently do not appear to be relevant or useful. FCMs can help students understand how seemingly disparate tasks fit within a larger structure. Eventually, understanding the dynamics of the FCM itself, as a model of a real system, may become the goal.
(2) *Provide information related to the task*—FCMs embed a large amount of information about a given domain. FCMs make evident what is and is not important within the context, reveal the categories which experts use to think about the system, and structure the domain in a graphical representation.
(3) Organizing and navigating complex information spaces has proven notoriously difficult.[35] FCMs may prove to be useful in organizing hypermedia environments. A simple system might use the FCM directly as a map of the information space, providing semantic links between conceptual nodes. A more advanced system might use a separate FCM to structure and organize the information adaptively. There is much room for research in this area.
(4) *Monitor trainee performance*—Sugrue and Clark[34] identify two methods for monitoring the external training environment: (a) data collection on aspects of trainees' performance and perceptions, and (b) guidance and tools to help trainees do their own monitoring or monitoring one another's performance.

The most obvious use of FCMs in this context is as a tool to enable students to monitor their own performance. Depending on the amount and timing of the monitoring required by the trainee, FCMs can be a just-in-time support, a central focus of support for practice. Utilizing appropriate group and individual processes, training systems can take advantage of the ease of generation and the feedback properties of FCMs to help trainees monitor the implications of their own models.

FCMs could also facilitate data collection for machine monitoring of student performance. Wallace and Mintzes[36] report that traditional concept mapping techniques are very sensitive to changes in students' conceptual frameworks. Further research will demonstrate whether this holds for FCMs as well. The FCM framework, however, gives developers the power to begin to compare student FCMs with expert FCMs. Two simple comparisons are measuring the centrality of a given node, and the effect, both direct and indirect, one node has on another. Kosko[19] has developed several methods for categorizing the importance and centrality of nodes within an FCM. A most simple measure is to sum the weights of the edges coming into and leading out of a given node.

Concept centrality: $\mathrm{CEN}(C_i) = \mathrm{IN}(C_i) + \mathrm{OUT}(C_i)$ where $\mathrm{IN}(C_i) = +e_{ki}$ (where $e_{ki}$ represents the causal edge function from $k$ to $i$), $\mathrm{OUT}(C_i) = +e_{ik}$ (where $e_{ik}$ represents the causal edge function from $i$ to $k$).

Nodes with high centrality and importance could be marked for special attention in data gathering or support of practice. The effect of one node on another can be measured with the following equations:

Indirect effect between concept $i$ and concept $j$:

$$I(C_i, C_j) = \min\left[e(C_p, C_{p+1}) \colon (p, p+1) E(i, k1, \ldots, kn1, j)\right]$$

where $e$: the edge function, $(i, k1, \ldots, kn1, j)$: the set of $i$ through $j$ edge functions, $p, p+1$: contiguous left-to-right path indices, total effect of concept $i$ on concept $j$: $T(C_i, C_j) = \max I_i(C_i, C_j)$.

The validity and usefulness of these measures has yet to be demonstrated, and more sophisticated techniques are sure to be developed.

## VIII. CONCLUSION

This paper has discussed the potential usefulness of fuzzy cognitive mapping in educational organization settings. The development of graphical tools to facilitate conceptual change is an important endeavor for educational technologists and facilitators of systemic change. By combining the capability of fuzzy logic to represent soft knowledge domains with dynamic modeling capabilities, the FCM framework has tremendous potential for contribution to the development of useful cognitive tools. FCMs are an extension of earlier concept mapping paradigms, yet they represent a significant advance over earlier, bivalent, static mapping systems.

There is much research to be done on the application of FCMs to education and instructional technology. It is heartening to know that there are a growing number of traditional engineering disciplines that are using FCMs to capture and represent expert knowledge resulting in the use of that knowledge in productive and meaningful ways. This paper will hopefully stimulate educational researchers to recognize the unique applicability of fuzzy logic to our field. As our understanding of the complexity of human learning increases, we must embrace new ways of describing, facilitating, and supporting that learning. Fuzzy cognitive maps represent one step in that direction.

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