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Mathematical modelling of eco-hydrological systems in the changing world

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Table of contents:

1 – Introduction to models

Modelling: replacing an object under consideration by a model, in order to draw information about the object from the model.

The model imitates (mimics) selected aspects of the (possibly very complex) real object of interest, which are deemed important in the study at hand, while for economy and clarity it should omit those deemed unessential.

Model is a working analogy of the real object (system). The basic rationale of modelling is the possibility to simulate and predict the behaviour of a complex object (system), with the help of a simpler, and/or more tractable, model.

Similarity but not identity!

Incomplete agreement of a model and a system, hence a model may give a distorted view at the system, and lead to false conclusions.

Classification of **formalized** models:

material models and **symbolic** models

Material models (representation of a real system by another real system):

physical (iconic, look-alike) models (e.g. hydraulic laboratory models of a dam or a channel, built in an appropriate scale) and **analogue** models (e.g. electrical analogs). Material models have similar properties to the object under consideration and are easier and cheaper to study. Experiments on material models can be made under more favourable and observable conditions, while experiments on the object may be impossible or difficult.

Symbolic models**:**

verbal models, **graphical** models, and **mathematical** models.

2 – Mathematical models

Nowadays, mathematical models are by far most commonly used, mainly as a result of the computational capabilities offered by affordable computers.

Mathematical modelling: use of mathematical constructs to describe features of systems or processes.

Virtually, every use of a mathematical equation to represent links between variables, or to mimic a temporal or spatial structure of variable(s) can be called mathematical modelling.

Stages in the process of mathematical modelling:

Selection of model structure;
Identification of values of model parameters
(calibration) [ill-posed, inverse problem];
Validation;
Use (simulation – hindcasting, forecasting);
Assessment.

Classifications of mathematical models:

Static – dynamic

Linear – nonlinear

Stationary – nonstationary

Deterministic – stochastic

Lumped – distributed

Physically-based – conceptual – black-box (system)

Continuous – discrete

Analytical - numerical

Criterion of classification – physical justification of models

* **Mechanistic, process-based** models expressing rigorous physical laws and theoretical concepts.
* **Black-box** (**system**) models, which match the input and output signals of the system, without mimicking the internal structure.
* **Conceptual** models, consisting of simple elements, which simulate, in an approximate way, processes occurring in the basin.

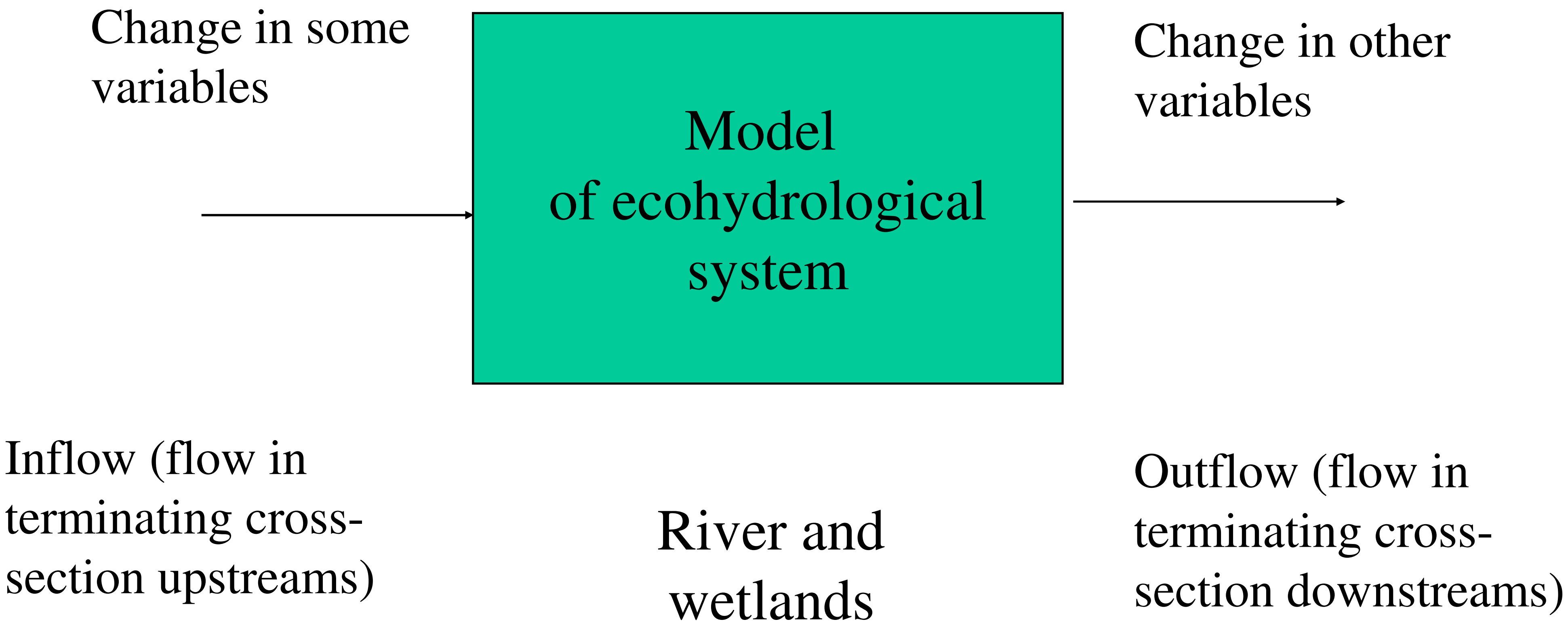
Ecohydrology links

ecology,

i.e. science on interrelationships of organisms and their environments,

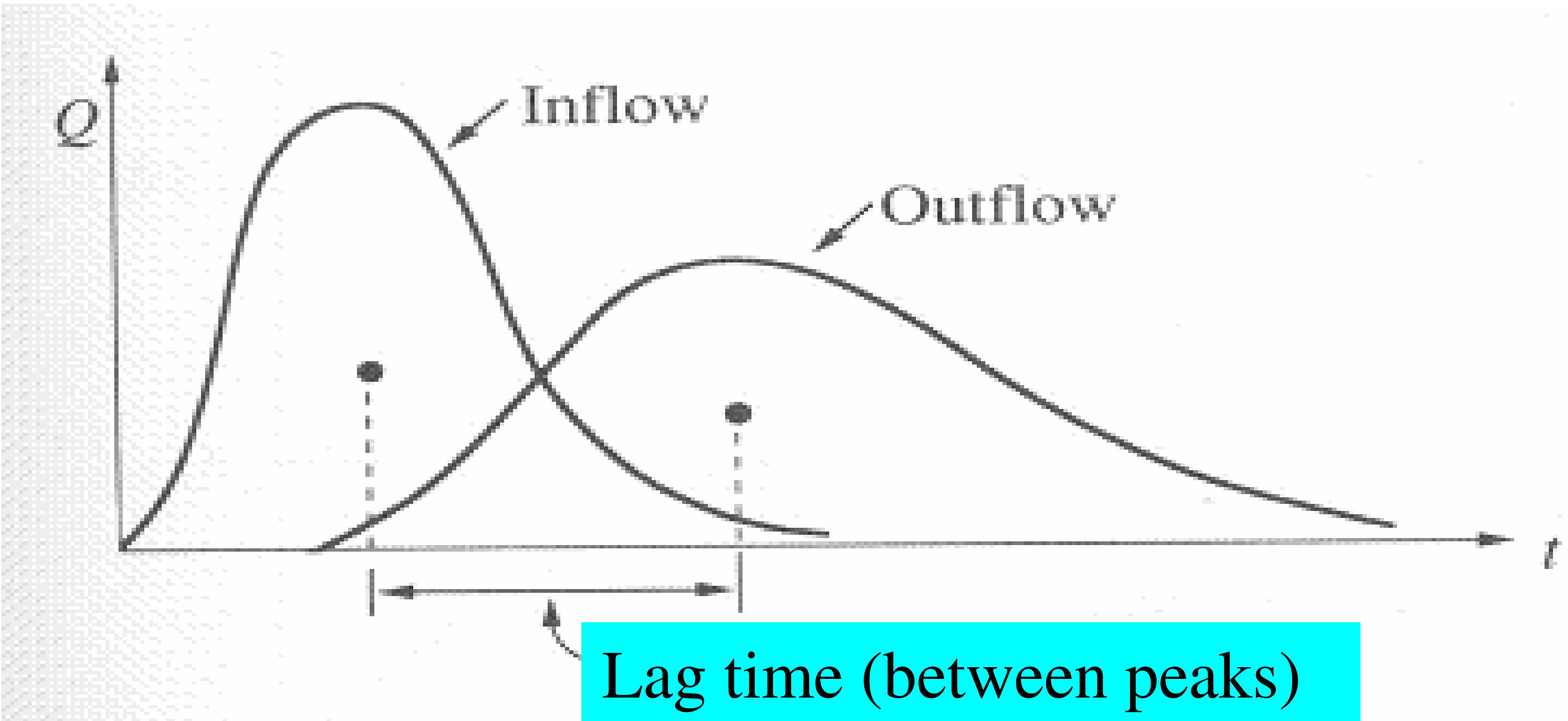
and **hydrology**,

i.e. science on water cycle in the nature, dealing with the properties, distribution, and circulation of water.


Change in some variables
Model of ecohydrological system
Change in other variables
Inflow (flow in terminating cross-section upstreams)
River and wetlands
Outflow (flow in terminating cross-section downstreams)

3 – Open channel flow: gallery of models to choose from

- Procedure to determine the outflow hydrograph from a river reach (downstream) from a known inflow hydrograph (upstream)



I can foretell the way of celestial bodies, but can say nothing on the movement of a small drop of water

[Galileo Galilei]

Flow routing in channels

- St. Venant equations (1870)

  – Continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

where Q is the flow rate, A is the cross-sectional area of flow

Momentum equation

Local acceleration term | Convective acceleration term | Pressure force term | Gravity force term | Friction force term

$$-\frac{1}{g}\frac{\partial V}{\partial t}-\frac{V}{g}\frac{\partial V}{\partial x}-\frac{\partial y}{\partial x}+S_o=S_f$$

Steady, uniform flow

Steady, non-uniform flow

Unsteady, non-uniform flow

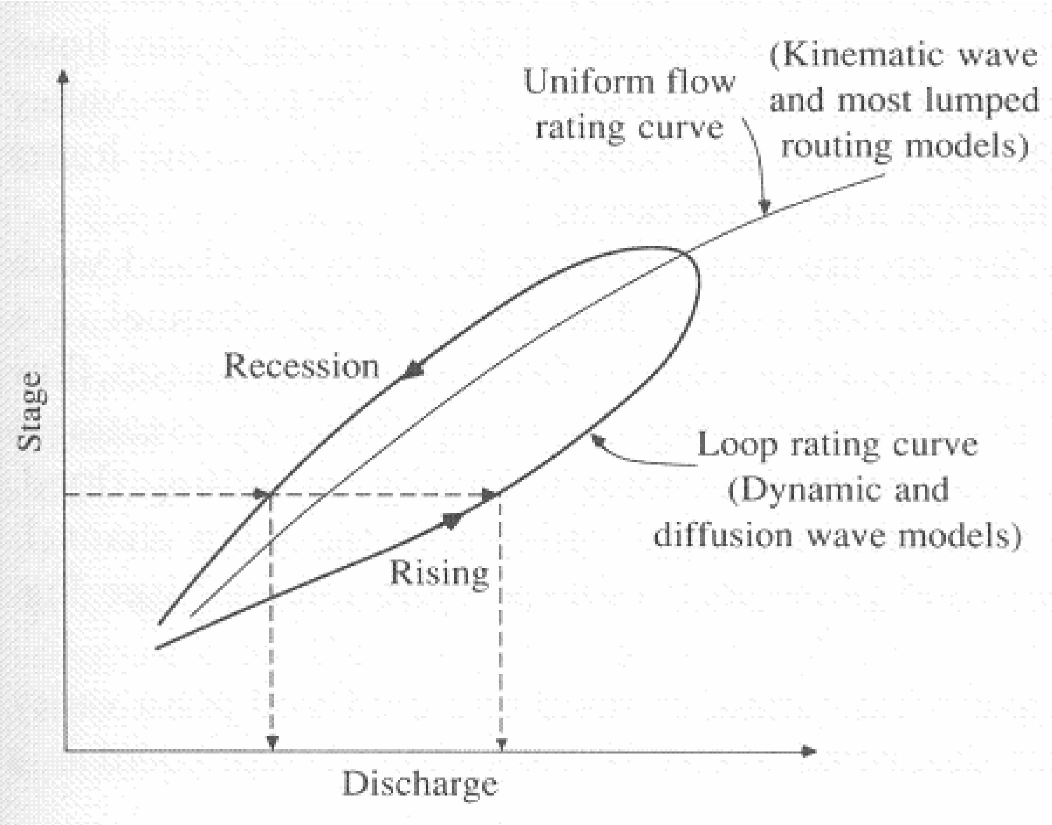
V- velocity, y – depth, S_o – bottom slope, S_f – friction slope

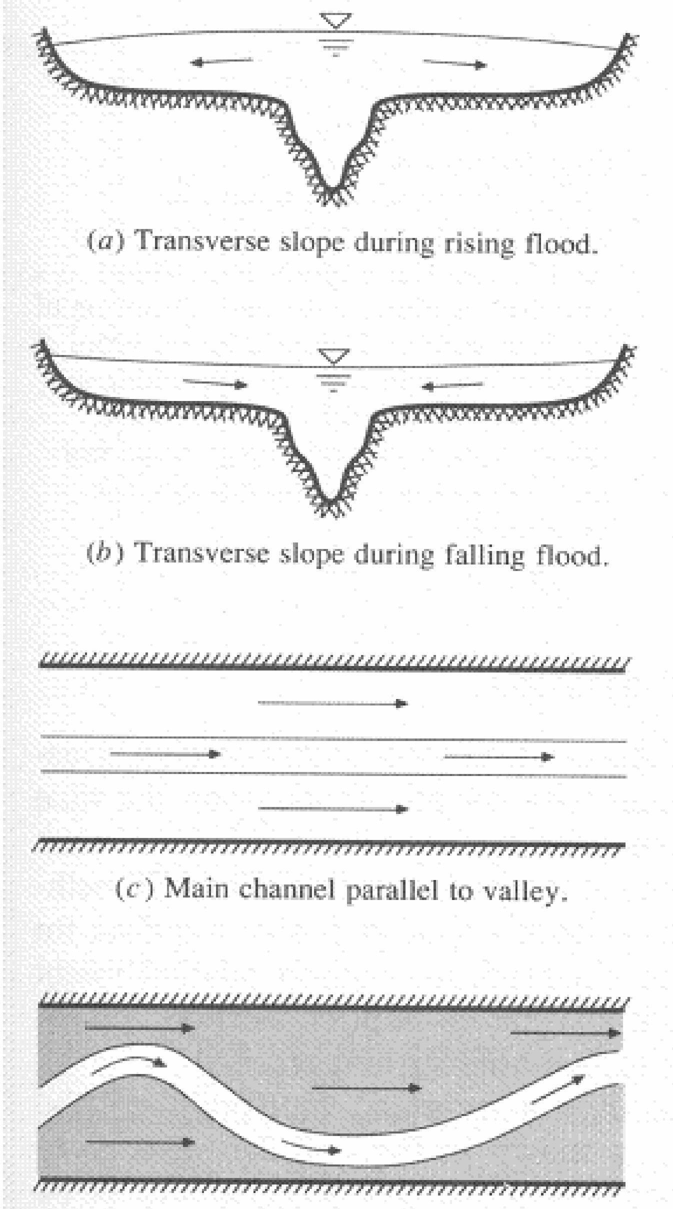
Assumptions for St. Venant equations

- Flow is one-dimensional.
- Hydrostatic pressure prevails and vertical accelerations are negligible.
- Streamline curvature is small.
- Bottom slope of the channel is small.
- Resistance law (e.g. Manning's) assumed.
- The fluid is incompressible.

…………………………..

Dynamic wave routing





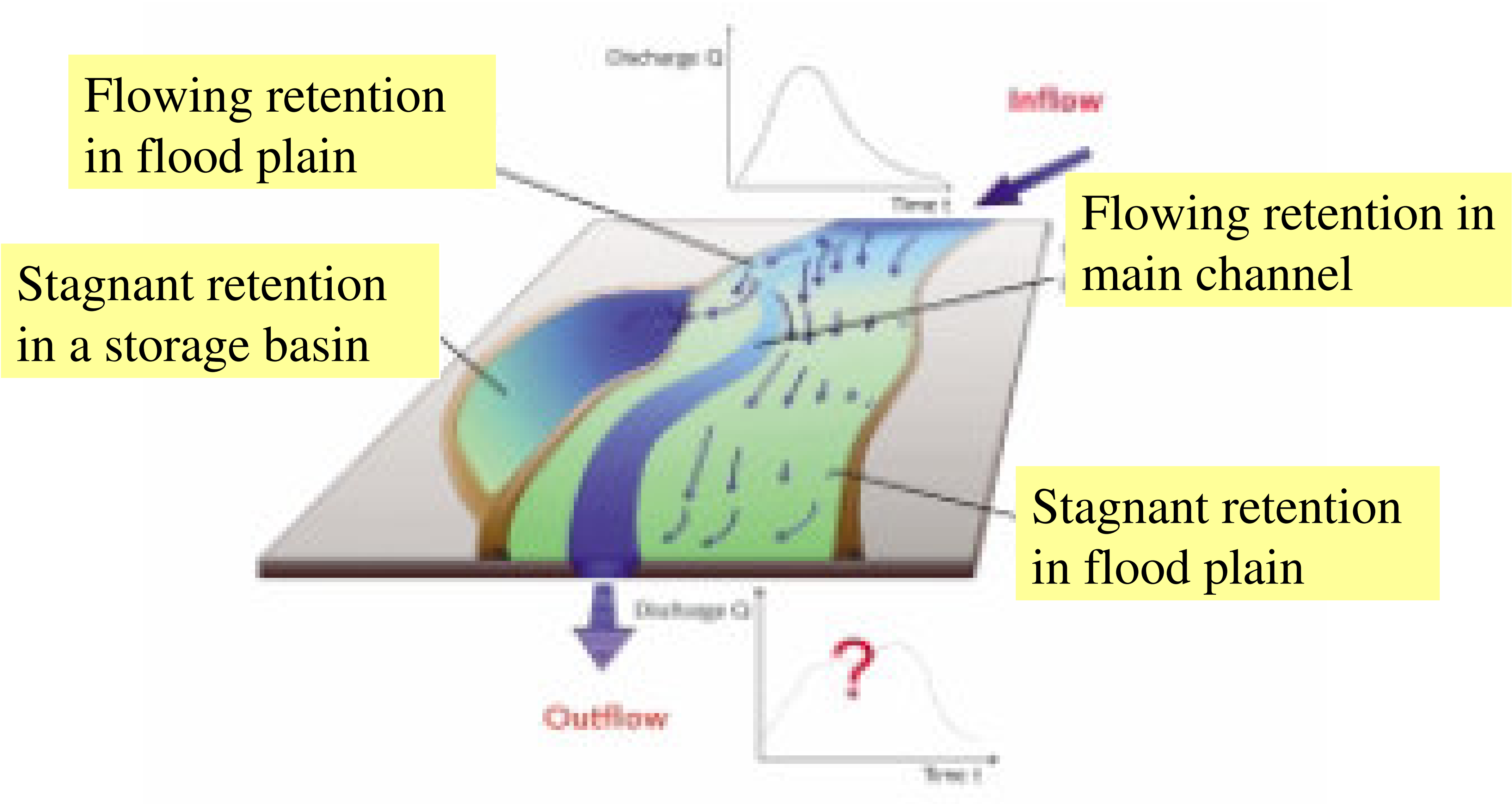
(*a*) Transverse slope during rising flood.

(*b*) Transverse slope during falling flood.

(*c*) Main channel parallel to valley.

(*d*) Meandering main channel.

Flow in natural channels is unsteady, nonuniform with junctions, tributaries, variable cross-sections, variable resistances, variable depths, etc etc.


Flowing retention in flood plain
Stagnant retention in a storage basin
Inflow
Flowing retention in main channel
Stagnant retention in flood plain
Outflow
?

Mathematics of a storage reservoir

Continuity equation

$$dS(t) / dt = I(t) - O(t),$$

where S(t) is the volume of stored water,

I(t) is the inflow to the reservoir (incl. precipitation),

O(t) is the outflow from the reservoir (incl. evapotranspiration, water abstraction, infiltration),

t is the time instant

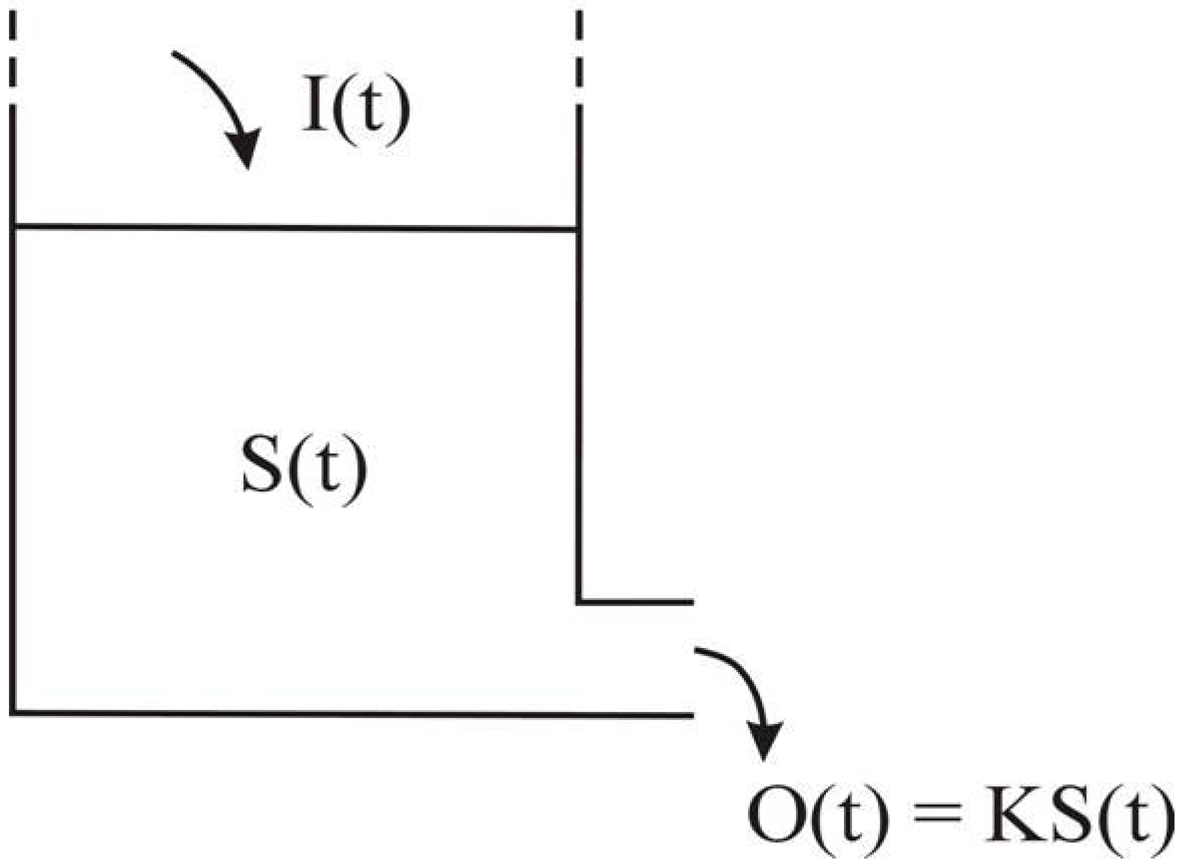
Outflow law

Linear

$$O(t) = K\ S(t)$$

$$O(t) = K(t)\ S(t)$$

Linear reservoir



Outflow law

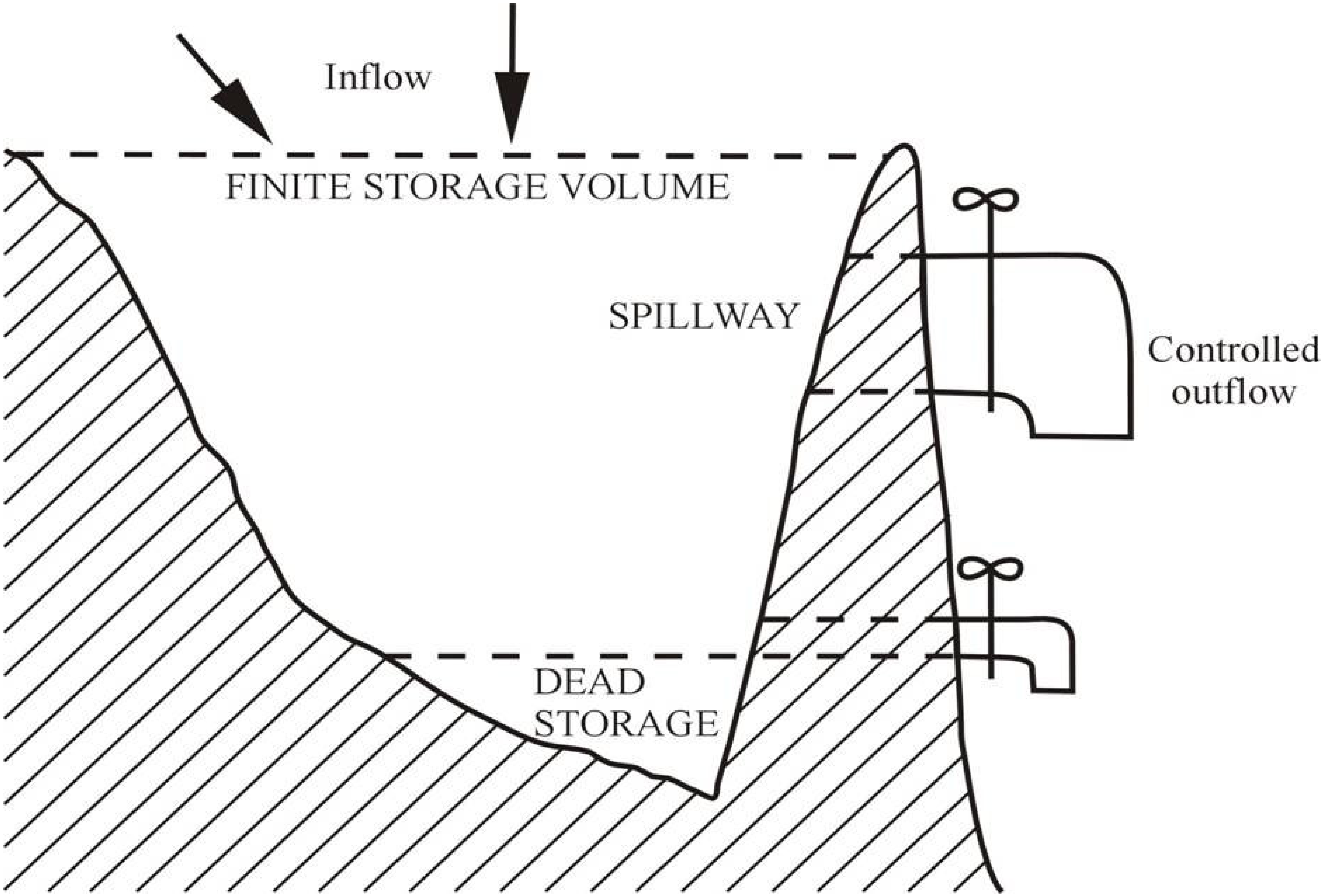
Nonlinear

$$O(t) = f[S(t)]$$

$$O(t) = K\, S^a(t)$$

$O(t) = 0$ for $S(t) < S_m$ and $K\,(S < S_m)$ for $S(t) > S_m$

$O(t) = K\, S(t)$ for $S(t) < S_M$ and $I(t)$ for $S(t) > S_M$


Inflow
FINITE STORAGE VOLUME
SPILLWAY
Controlled
outflow
DEAD
STORAGE

The convolution integral allows one to find the output of a linear system corresponding to any input.

$$y(t) = \int_0^t x(t - \tau)\, h(\tau)\, d\tau$$

$$y(t) = \int_0^\infty x(t - \tau)\, h(\tau)\, d\tau$$

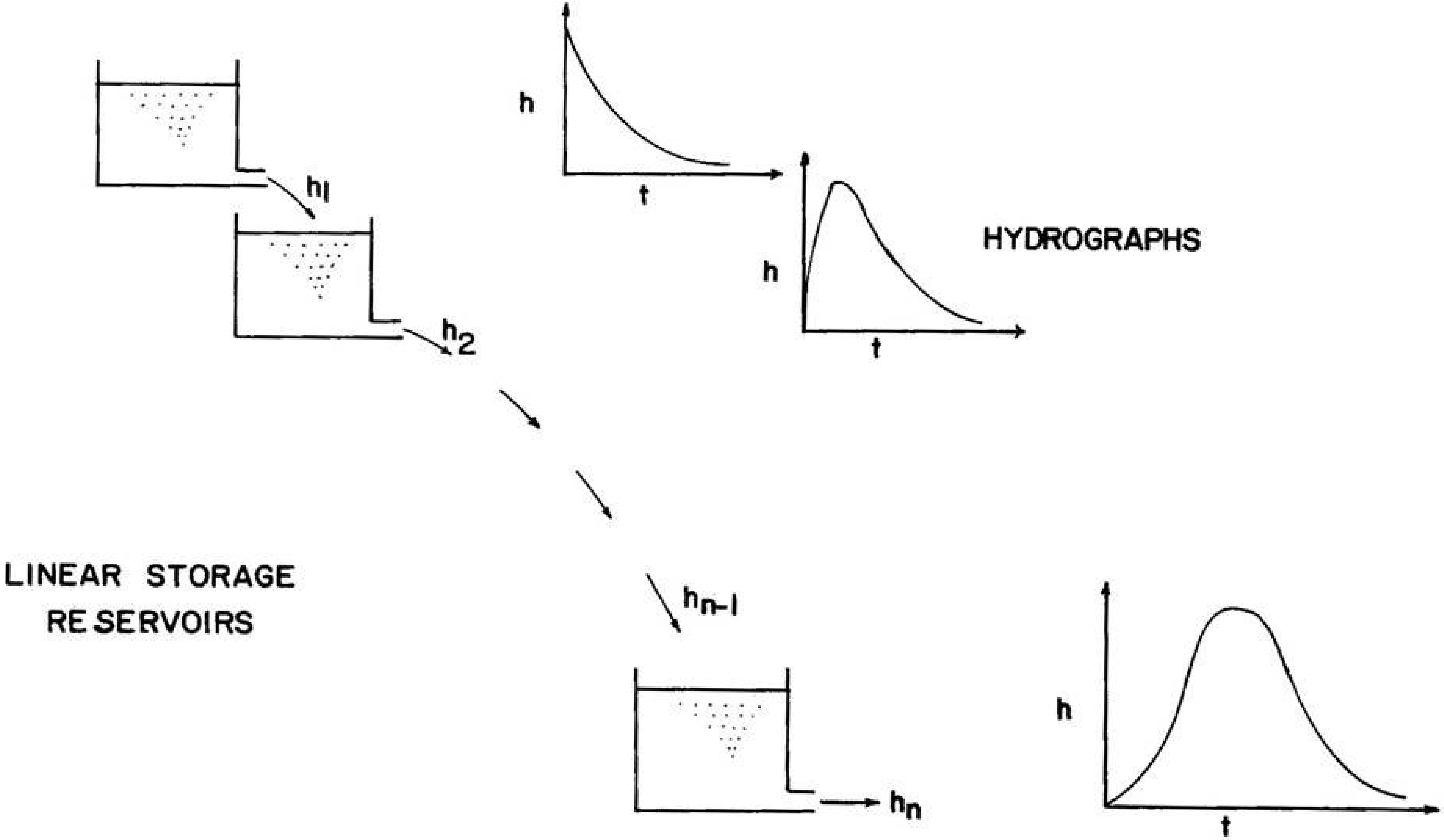
$$y(t) = \int_0^T x(t - \tau)\, h(\tau)\, d\tau$$

Properties of a kernel function (impulse response) of a conservative system:

$h(t) \rightarrow 0$ for $t \rightarrow \infty$

$h(t) \geq 0$ and $|h(t)| \leq M$ for all t

$$\int_0^\infty h(t)\,dt = 1$$



h
t
h
t
HYDROGRAPHS
h₁
h_2
LINEAR STORAGE
RESERVOIRS
h_n-1
h_n
h
t

Linear Saint Venant equation:

$(g\, y_o - v_o^2)\, \partial^2 Q/\partial x^2 - 2\, v_o\, \partial^2 Q/\partial x \partial t - \partial^2 Q/\partial t^2 - 3\, g\, S_o\, \partial Q/\partial X - \partial Q/\partial t = 0$

for a semi-infinite, prismatic river reach with rectangular cross-section

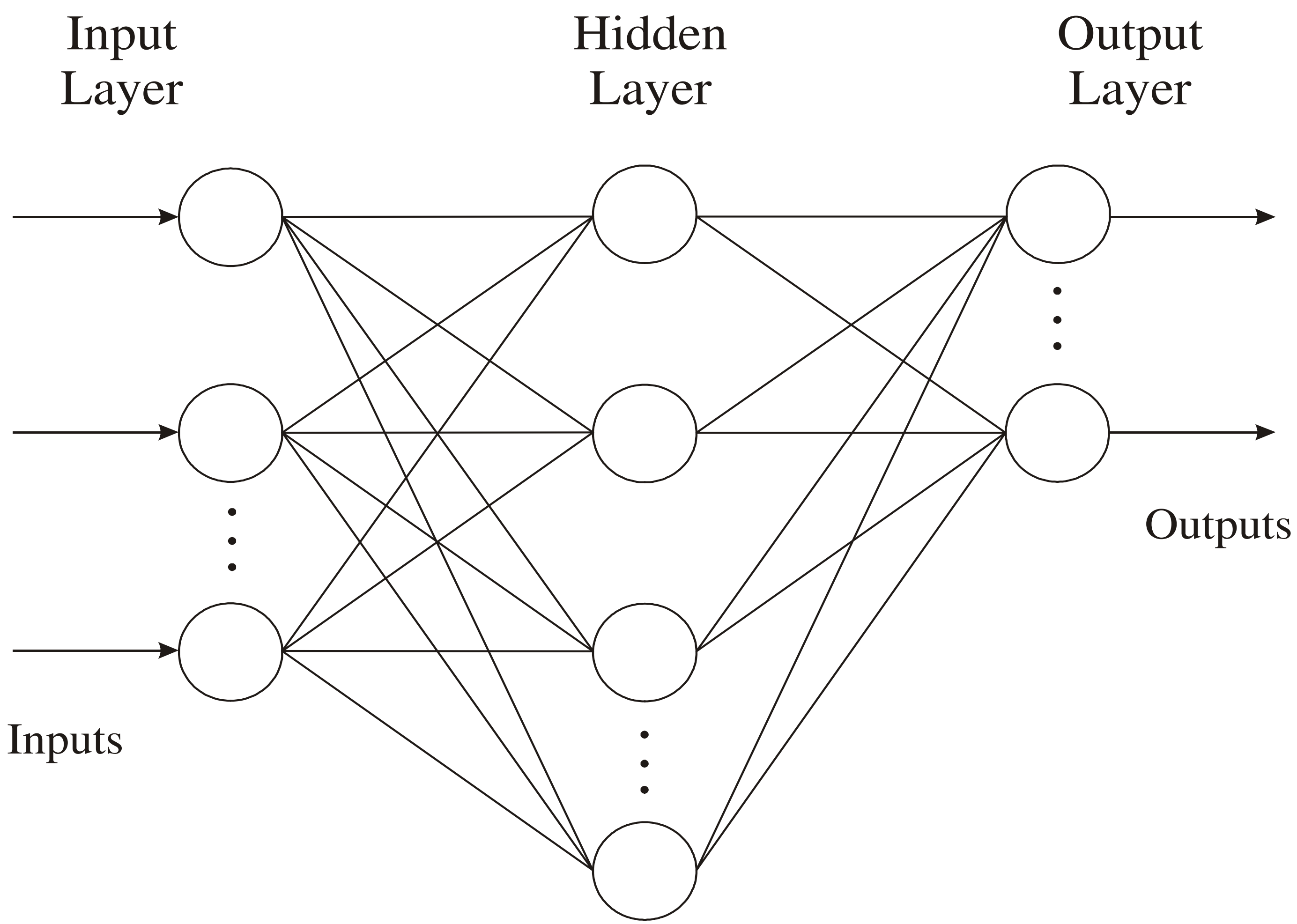
h(t) is very complex

Artificial neural networks and fuzzy set models

-Nonlinear models capable of representing a wide range of complex relationships

-Equation-free universal estimators trained from observed data; largely automatic, with no need for manual training

Artificial neural networks have been used to model diverse hydrological proceses, therein open channel flow.



ANNs:

Black-box approach (no insight into the physics)

Computer time

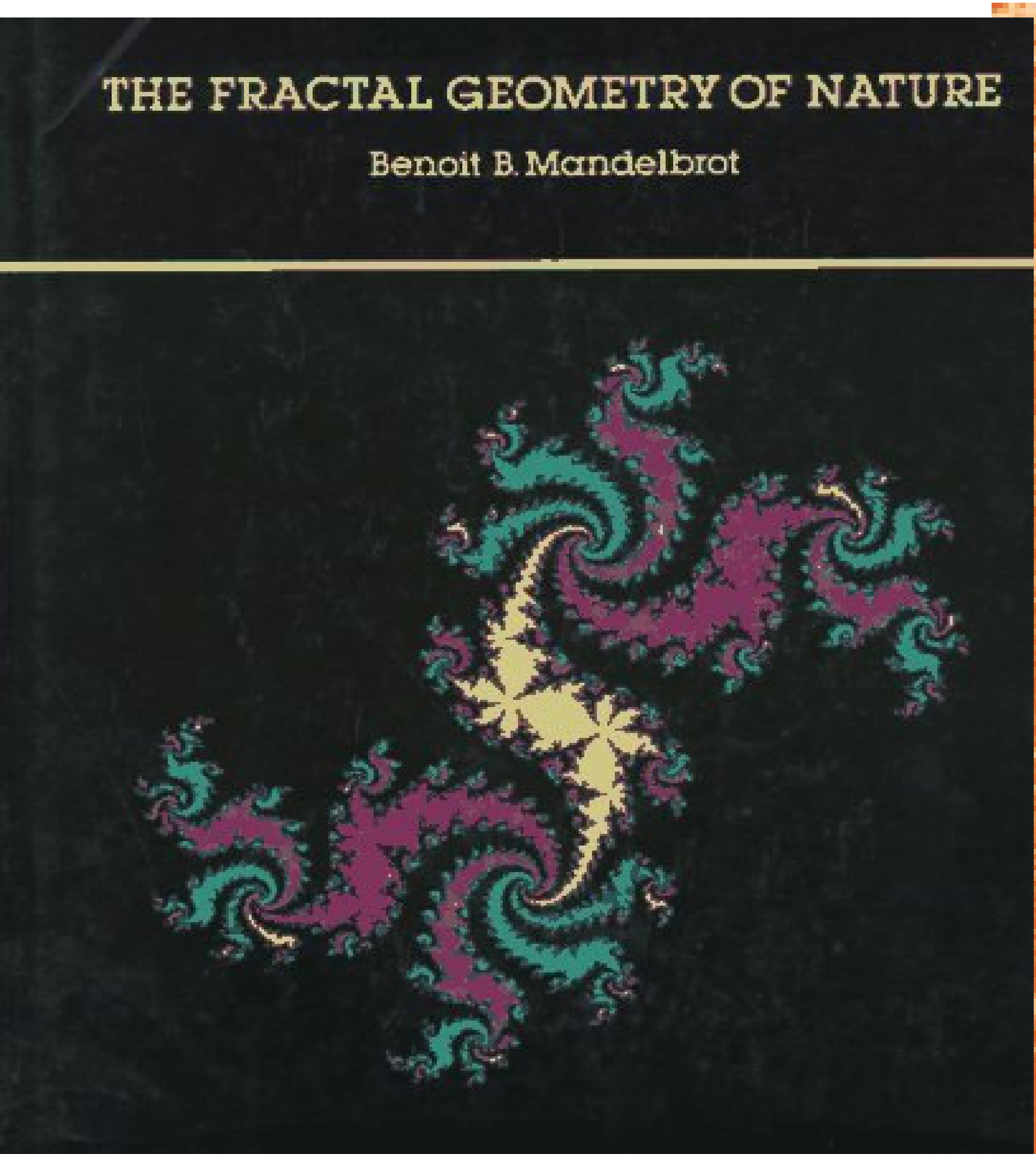
Problems with training

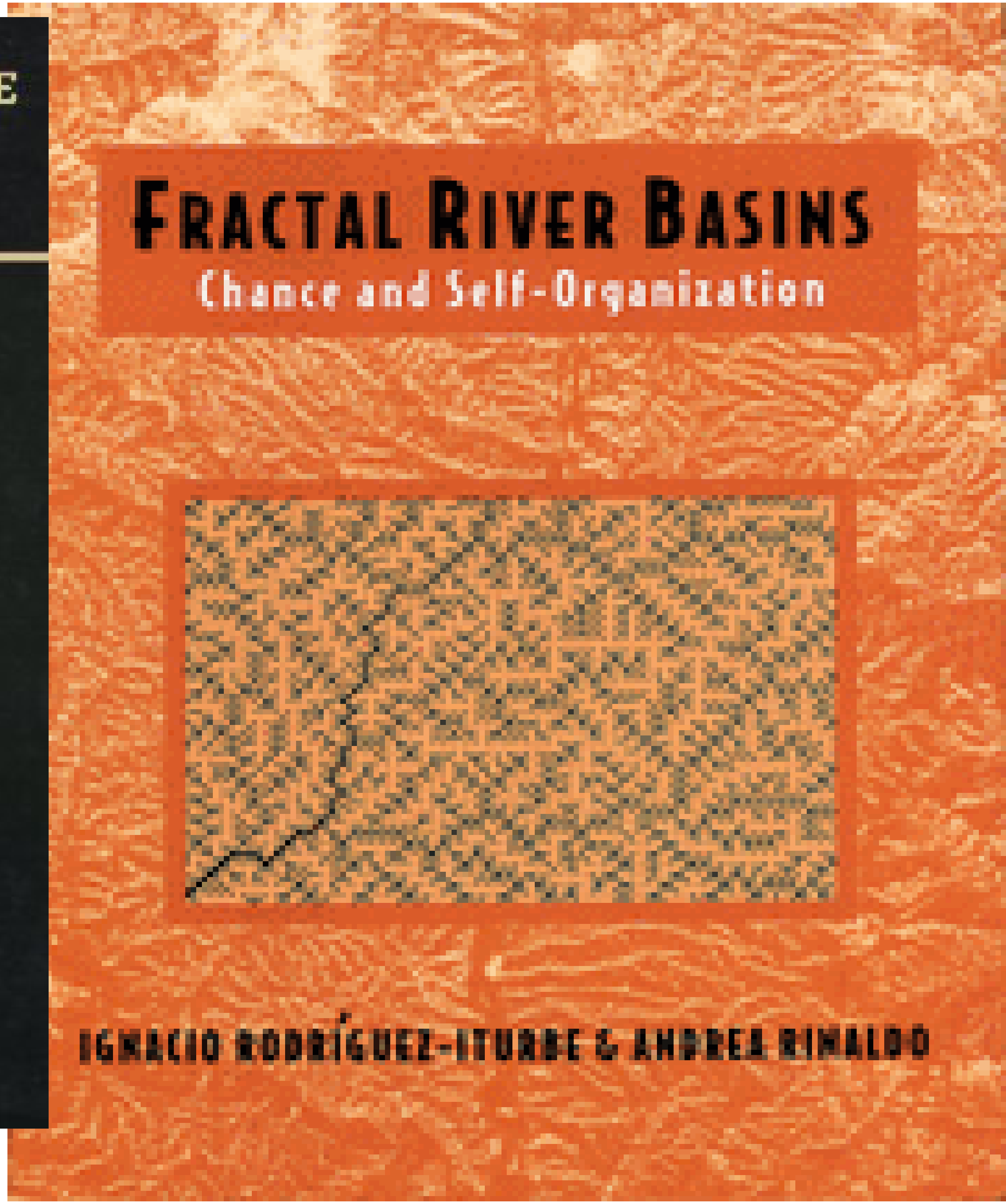
May not always work

Fuzzy sets approaches

Provide a framework for the interpretation of any existing theoretical knowledge and adding to it a learning process (less black-box).

Fuzzification – solution - defuzzification


THE FRACTAL GEOMETRY OF NATURE
Benoit B. Mandelbrot


FRACTAL RIVER BASINS
Chance and Self-Organization
IGNACIO RODRÍGUEZ-ITURBE & ANDREA RINALDO

Genetic algorithm mimics the biological evolution based on the Darwinist theory (survival of the fittest), where the strongest (or any selected) **offspring** in a **generation** are more likely to **survive** and **reproduce**. The method starts with a number of possible solutions, referred to as the first generation of the population. Each of the possible solutions is referred to as an individual, then encoded as either binary or real-coded string (called **chromosome**). For each individual, the objective function is evaluated. During the course of the search, new generations of individuals are reproduced from the old generations through random **selection**, **crossover**, and **mutation** based on certain probabilistic rules. The selection is in favor of those interim solutions with lower objective function values (in a minimization problem). Gradually, the population evolves toward the optimal solution.

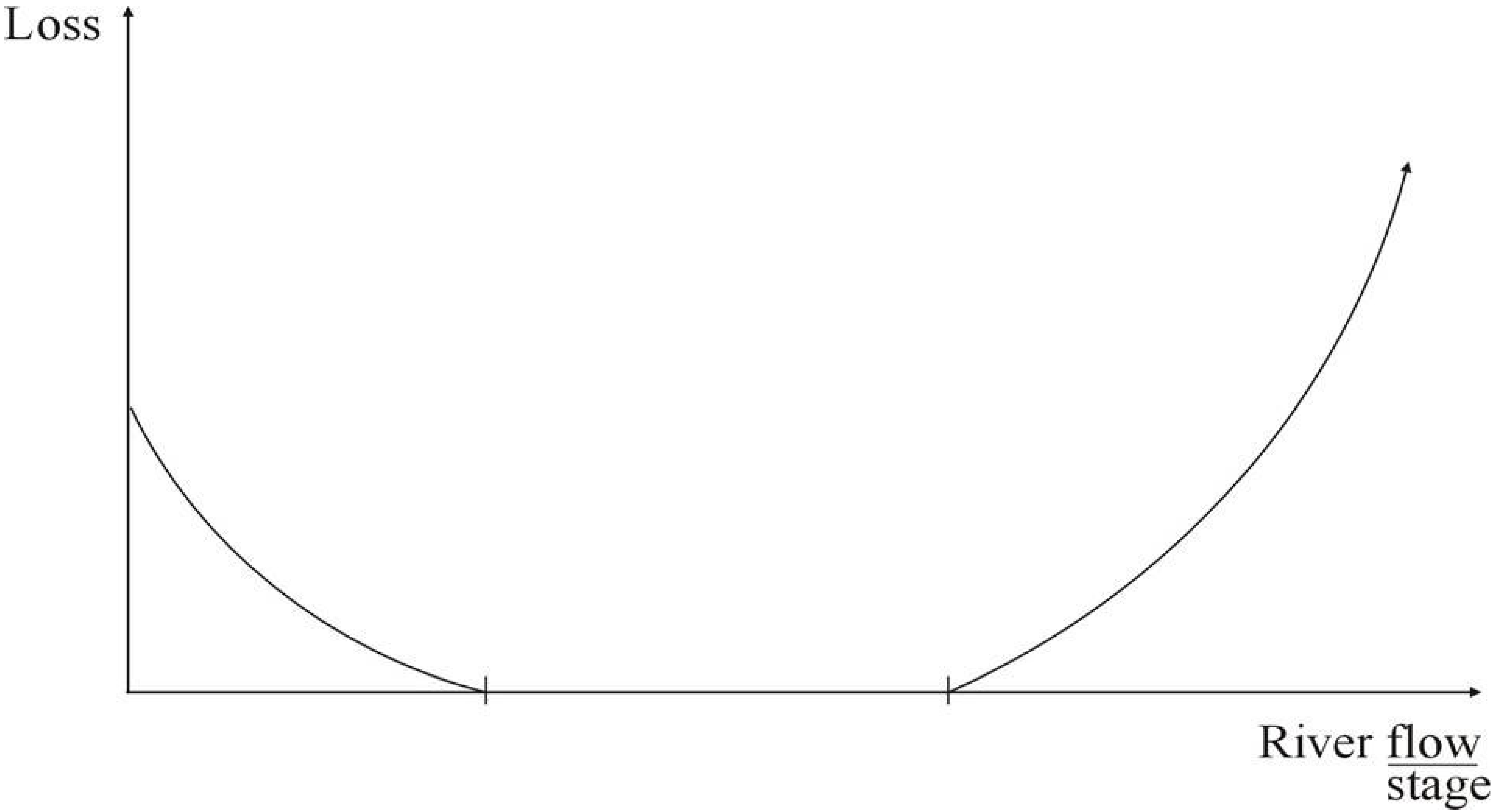
4 – More than water!

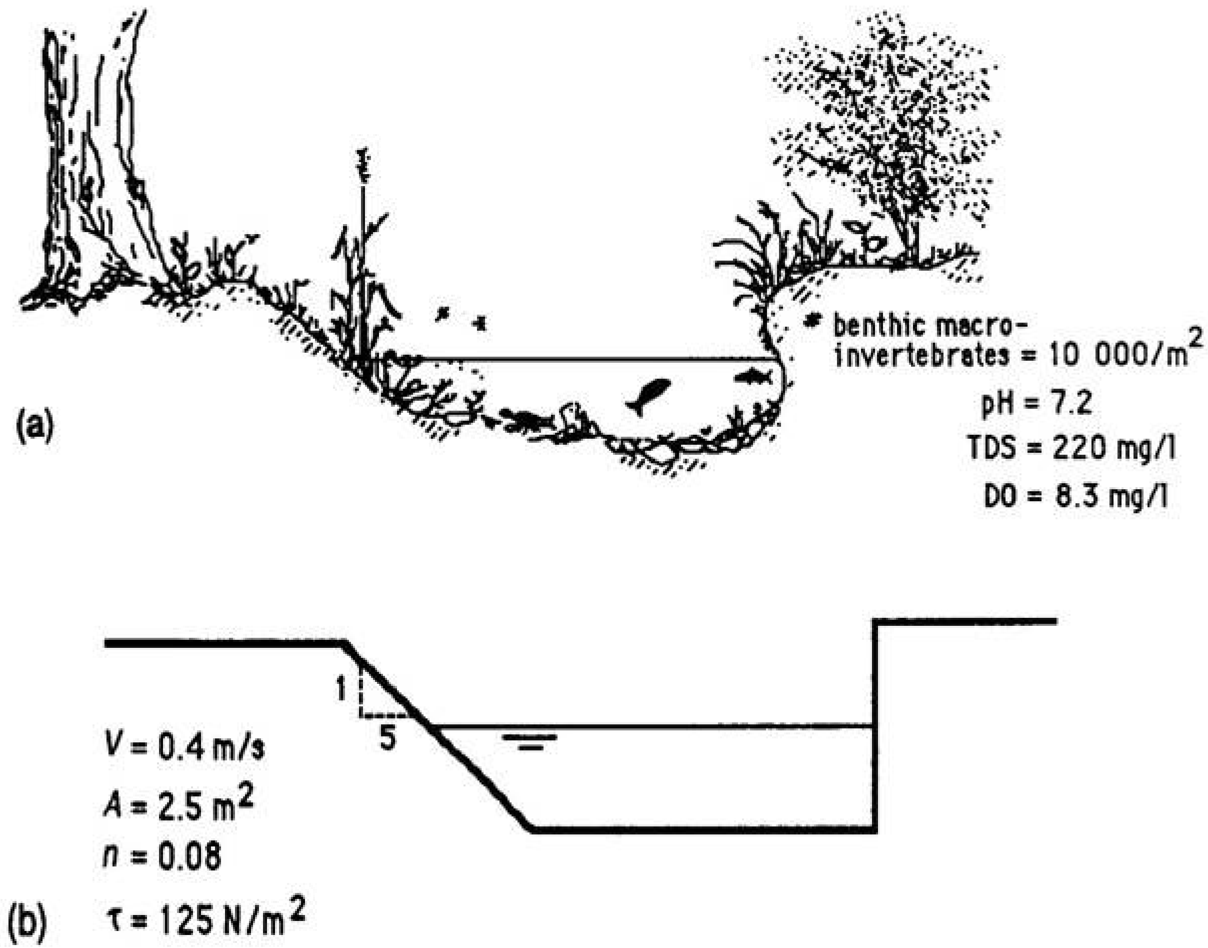
Sample of themes from the morning session:

Vegetation growth vs water levels and river flows
Water quantity and quality interaction in wetlands (DUFLOW model)
Material fluxes, sedimentation, willow plantation design (NCCHE2D, U. of Mississippi)
Re-use of drainage water – passive in-stream wetland treatment (HEC-RAS and MATLAB)
Flow routing in wetlands (GPS, DEM, Landsat imagery)
Simulating managed floods

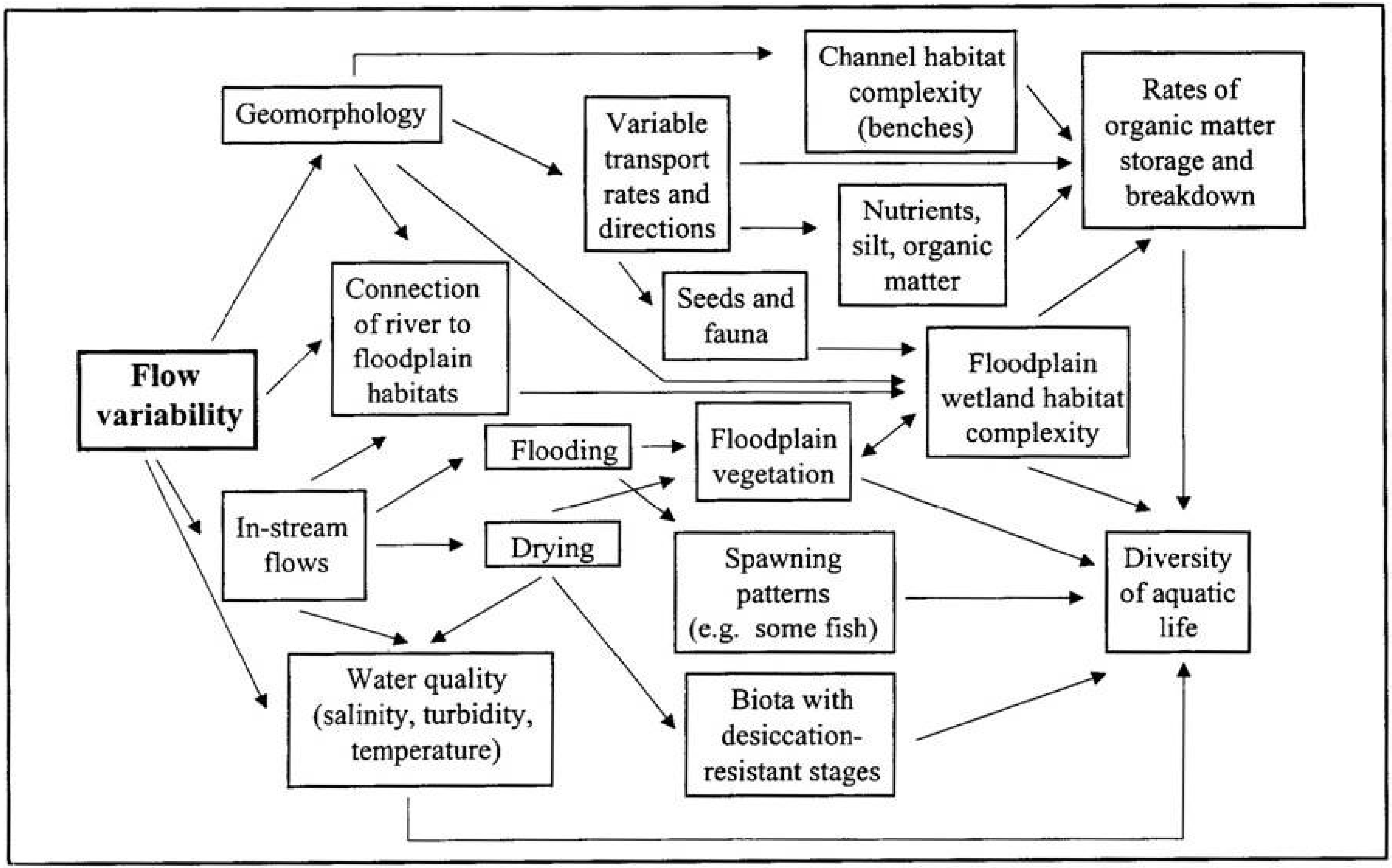
Apart from **climatic/meteorologic** input (precipitation, temperature, etc.) and **water-related** components (water quantity: level of surface or ground water, water flow, soil moisture; and water quality), a mathematical model of an ecohydrological system should deal with **biotic** elements (e. g. bilateral links between water and vegetation, and biodiversity) and possibly with **socio-economy** (e. g. ecosystem services).

Loss function – not easy to get!



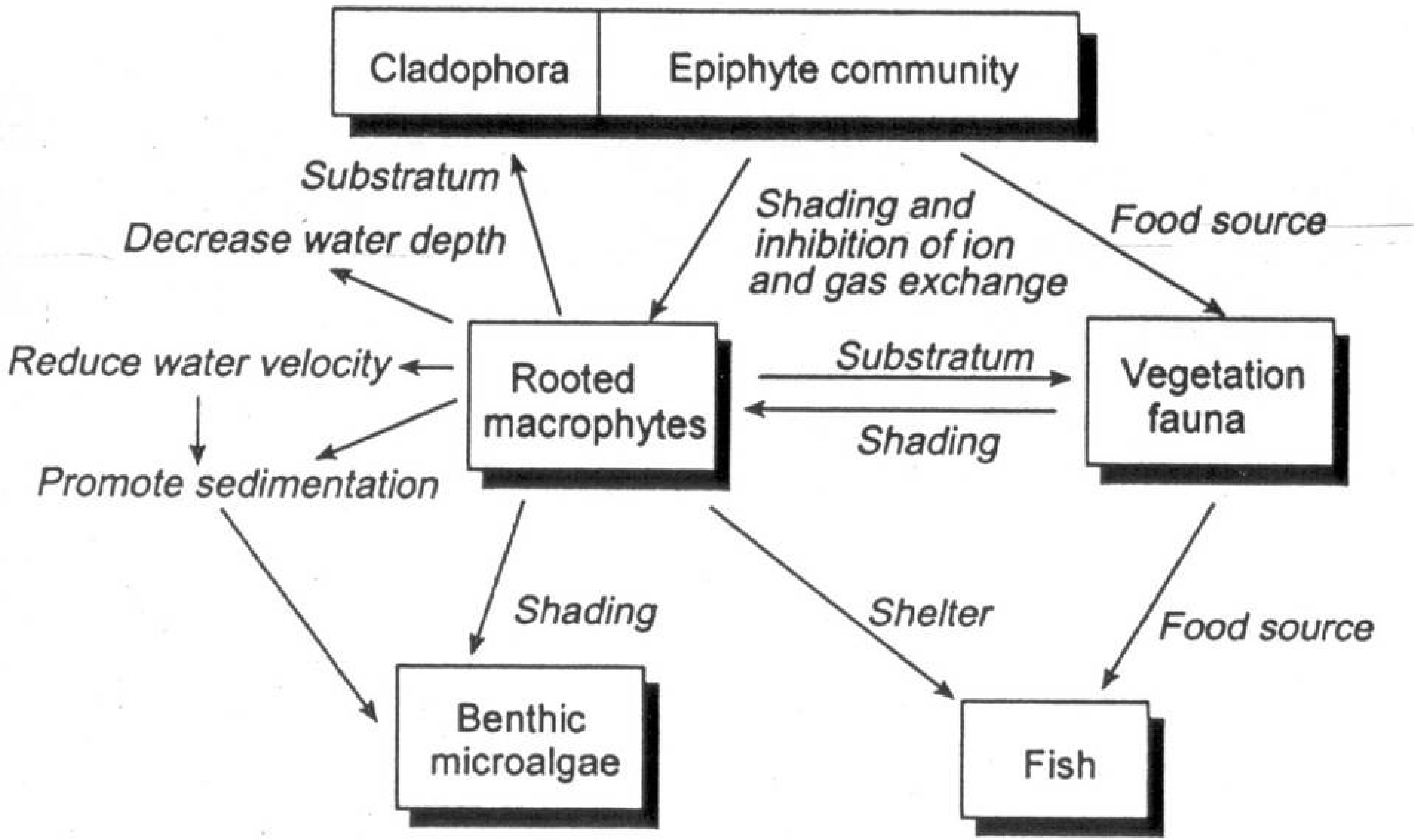


Ecologist's and engineering hydrologist's view of a stream (from Gordon *et al*.)



Importance of flow variability to biota and ecosystem processes in rivers (simplified scheme).

From Boulton *et al.* in Boon *et al.*

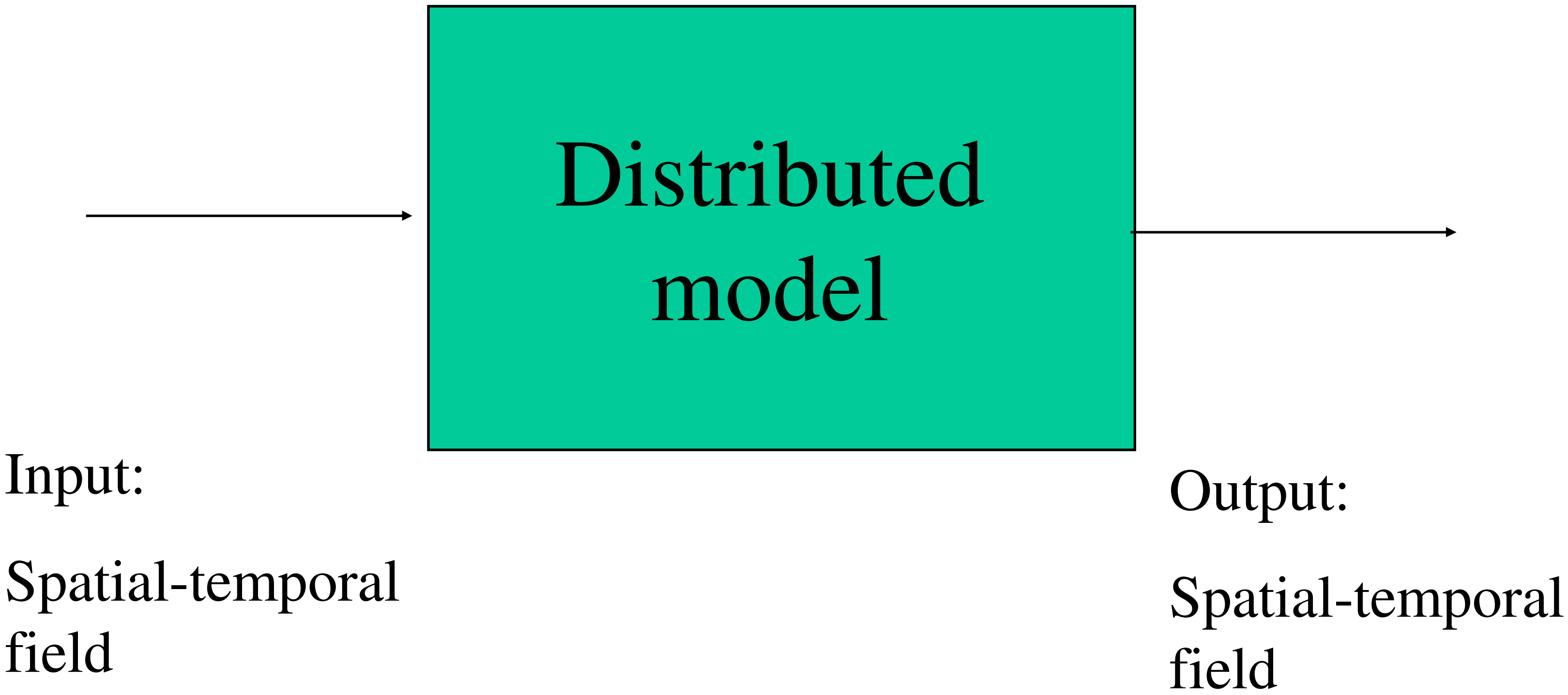


The roles played by rooted macrophytes in lowland streams (after Large & Prach in Baird & Wilby)

Developments in the modelling of eco-hydrological systems have been linked closely with the emergence and progress of electronic computers, user-friendly operation systems, and application software. The ubiquitous availability of inexpensive computers and the development of associated numerical methods makes it possible to carry out complex, repetitive calculations that use large quantities of data (barriers: data availability, understanding).

Distributed hydrological models are getting increasingly common, in order to make use of the distributed data fields, which become available from remote sensing, and benefit of the GIS revolution.

Google: 440,000 entries to „distributed hydrological model"



Distributed models require a large amount of data, which often either do not exist or are not available.

Numerous parameters of a distributed model cannot be measured in the field and calibration of a such a model is a difficult optimisation task.

More complex and physically-justified descriptions cannot be implemented, because they would require more parameters, which need to be identified.

Physically-based distributed models use small-scale equations with an assumption that the change of scales can be accommodated by the use of “effective” parameter values.

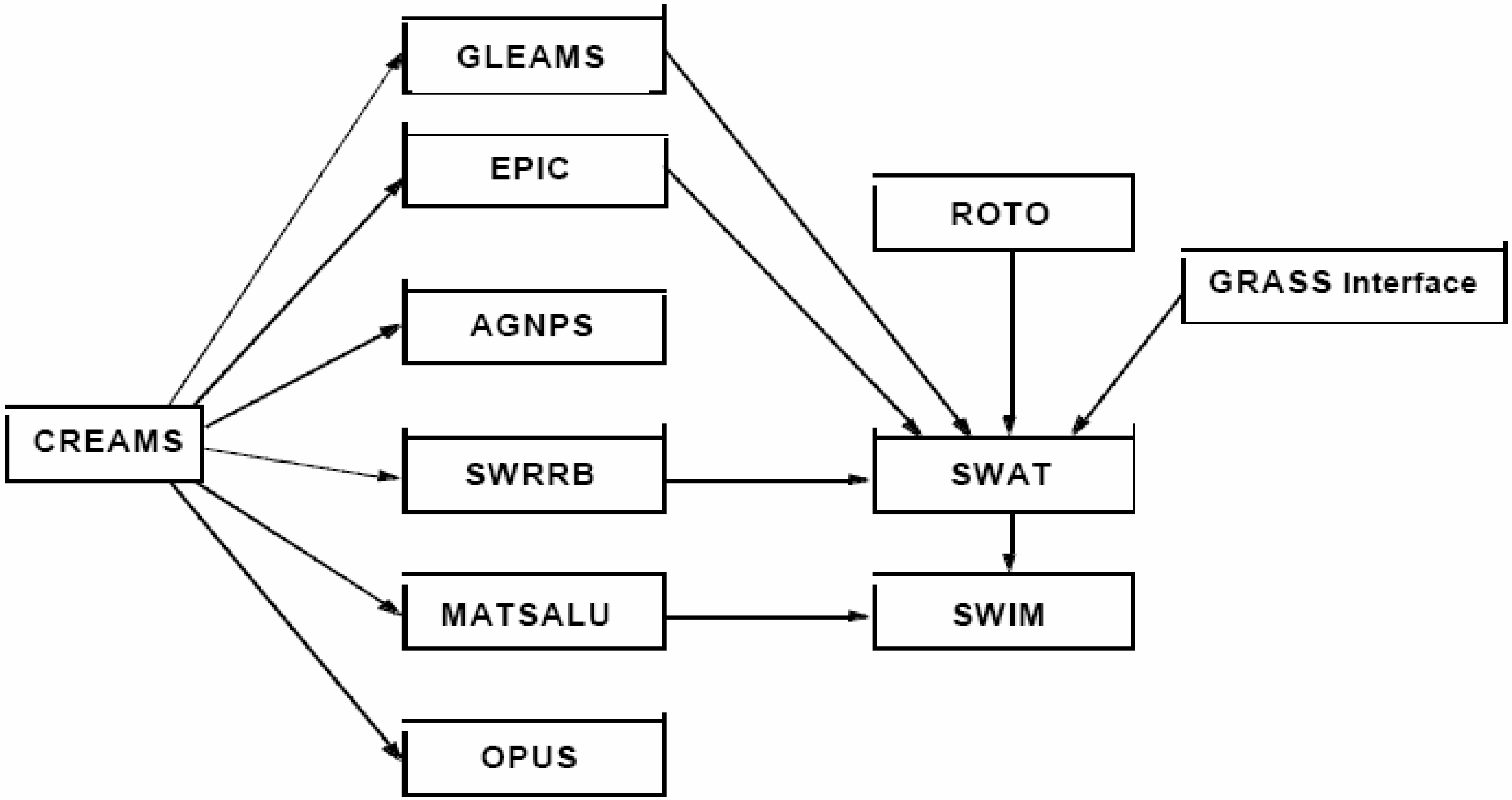
Example:

SWIM (Soil and Water Integrated Model)

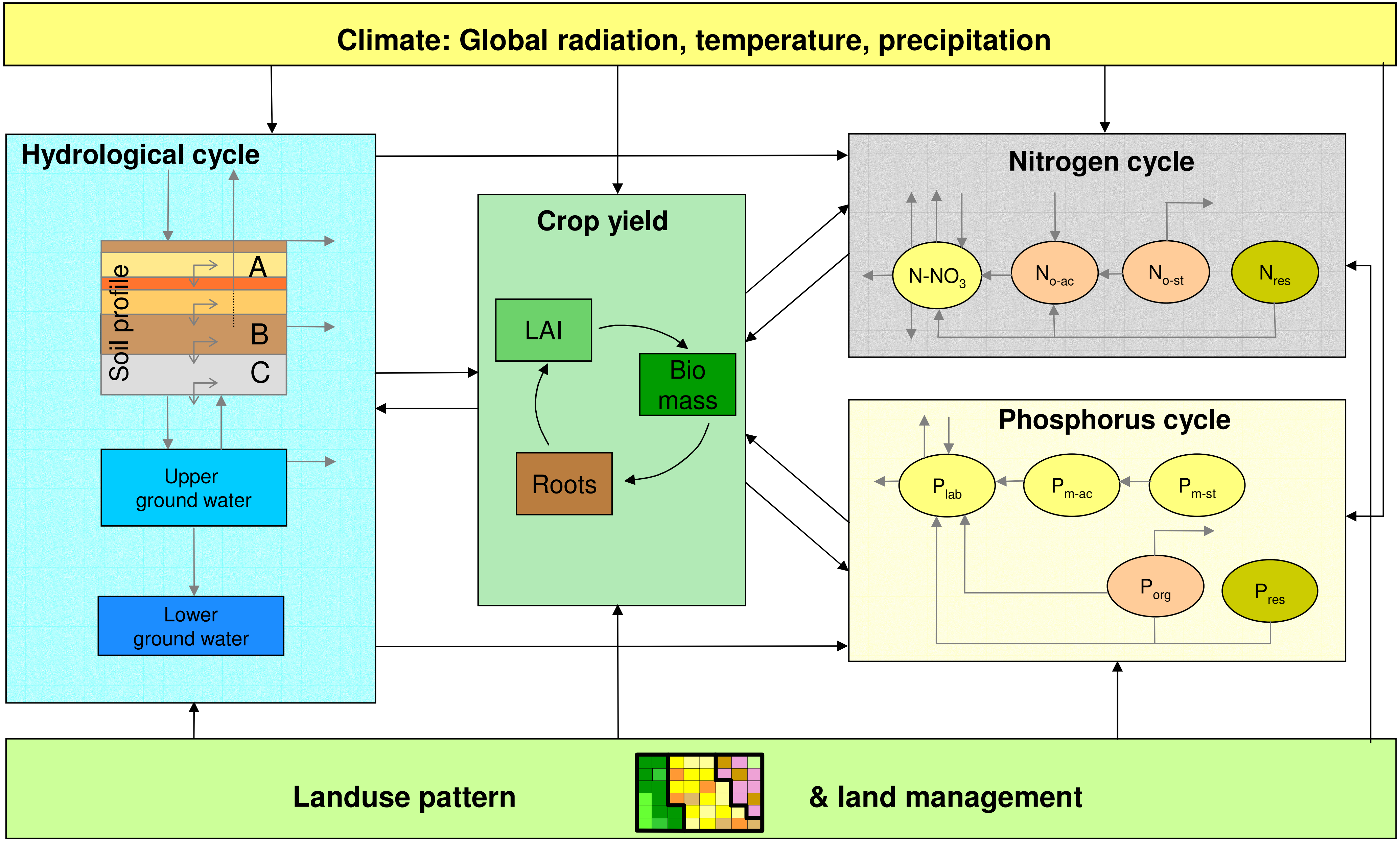
Distributed model

Author: Dr Valentina Krysanova et al. (PIK)

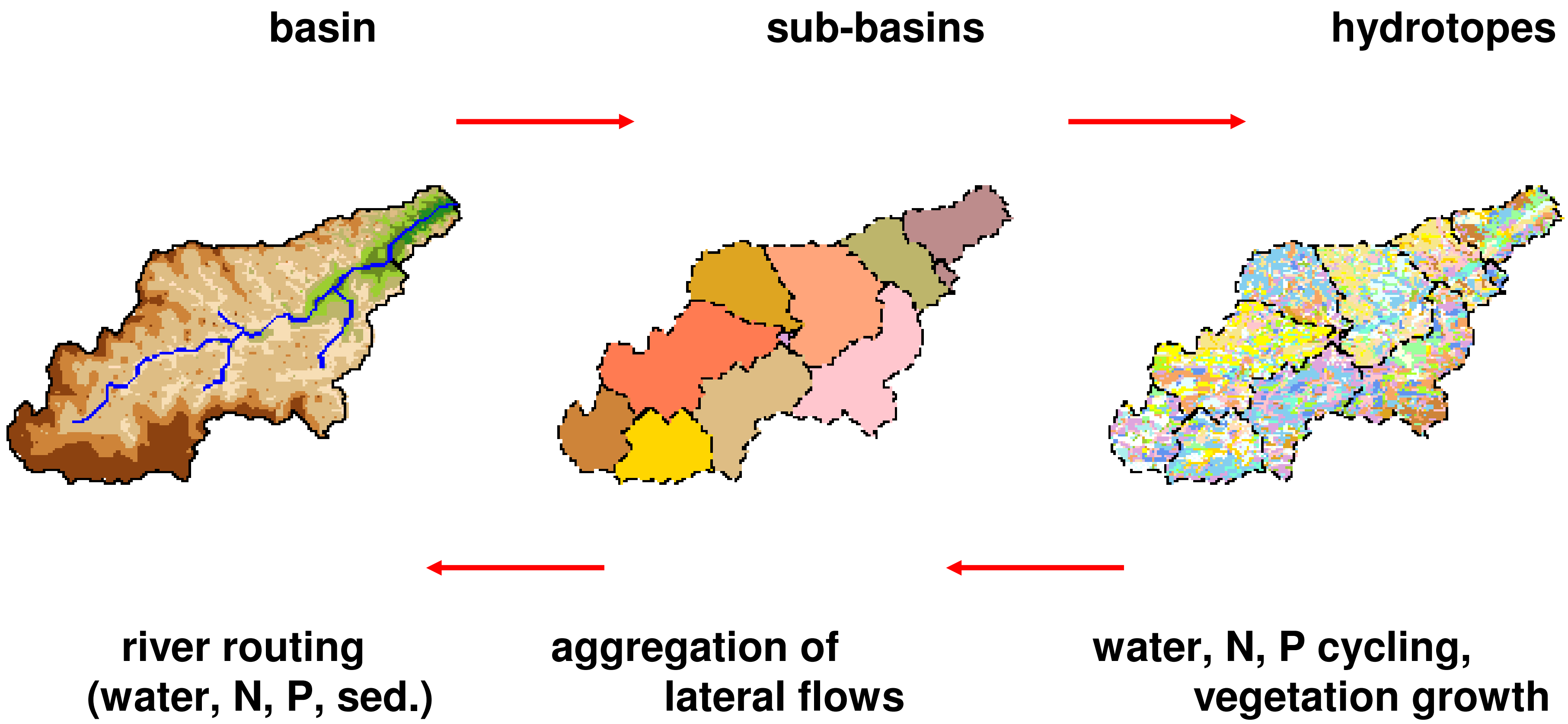
Genealogy of SWIM



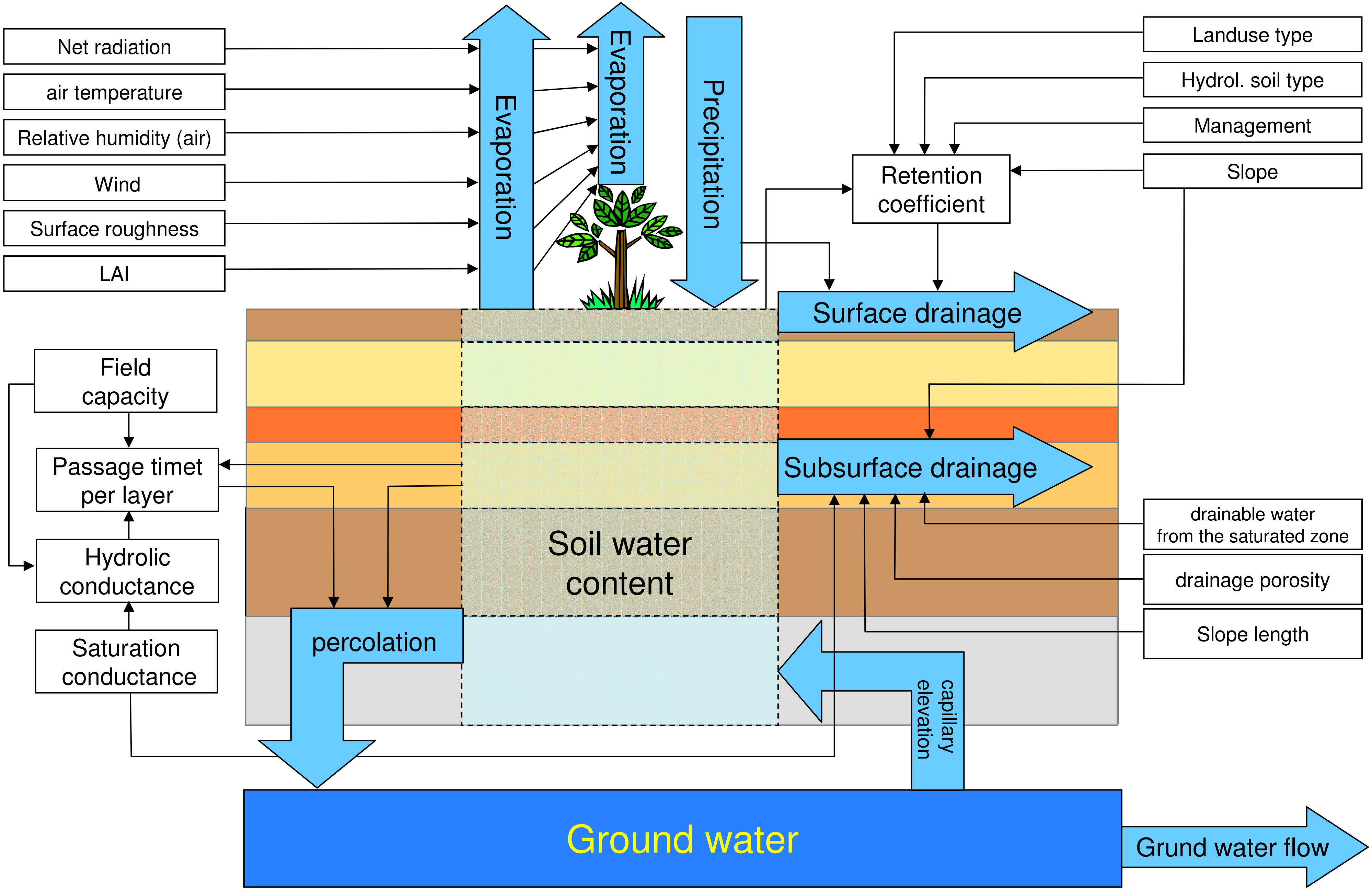
SWIM (Soil and Water Integrated Model) overview



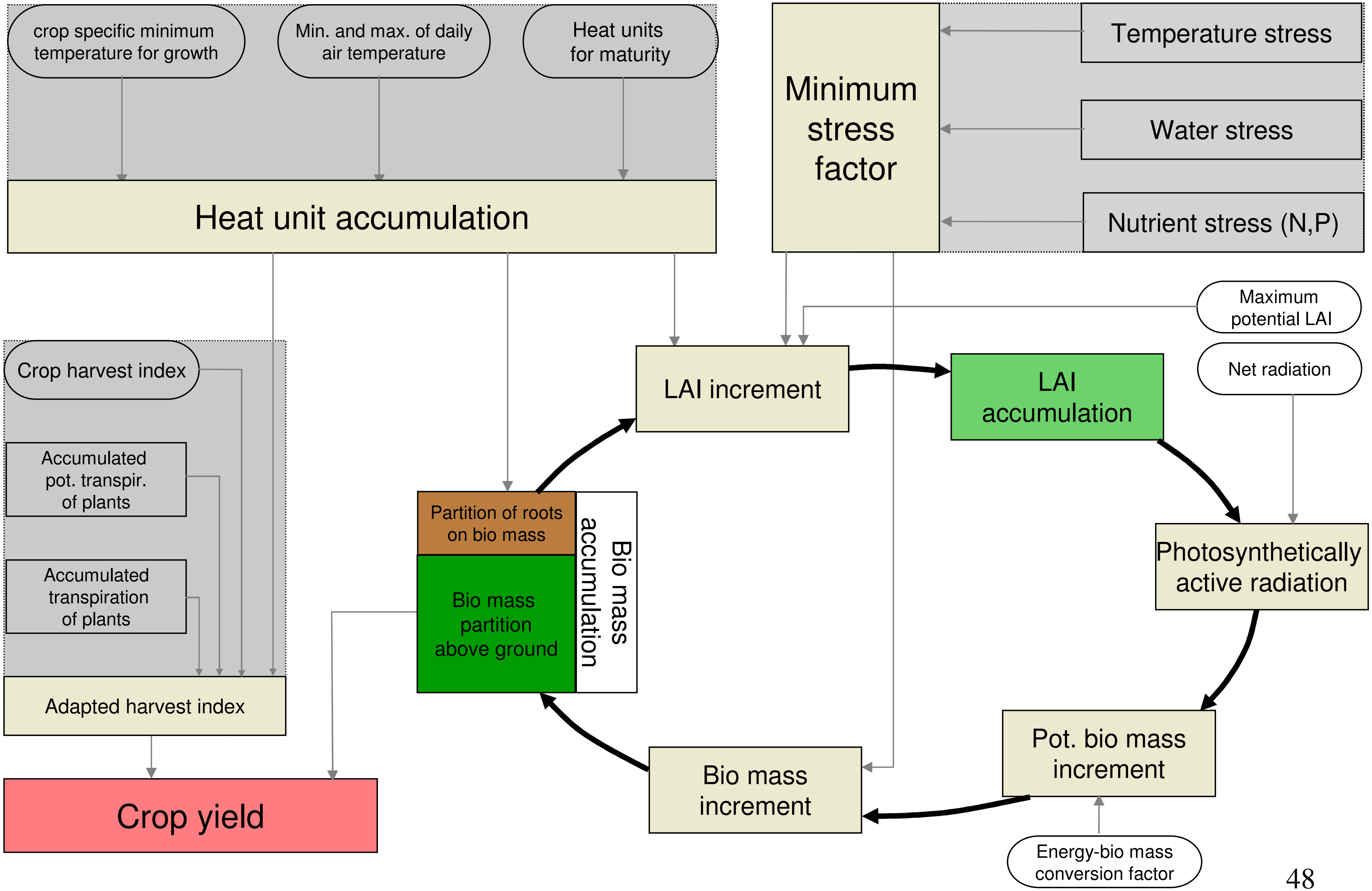
Spatial disaggregation in SWIM



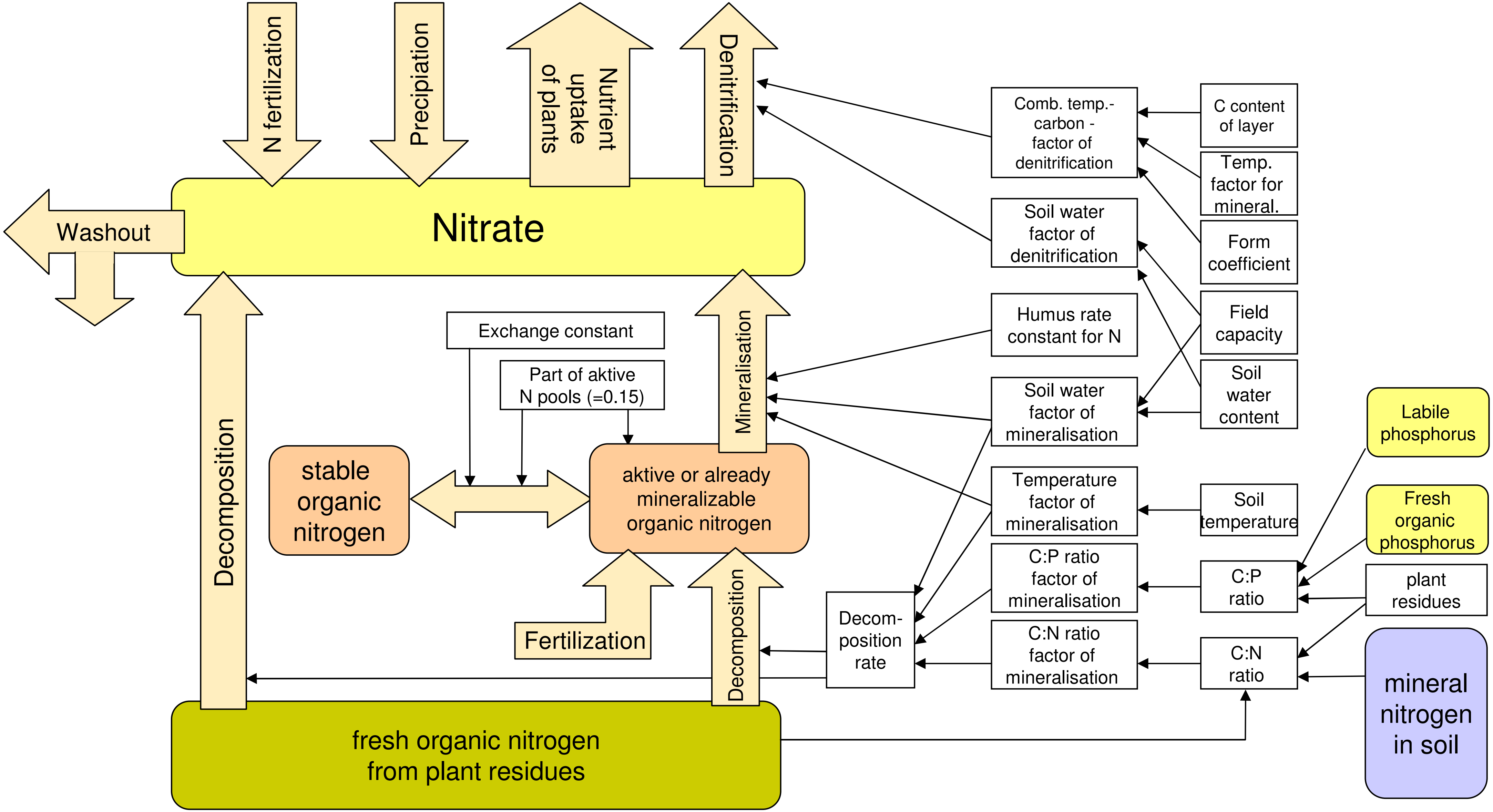
Hydrological cycle in SWIM



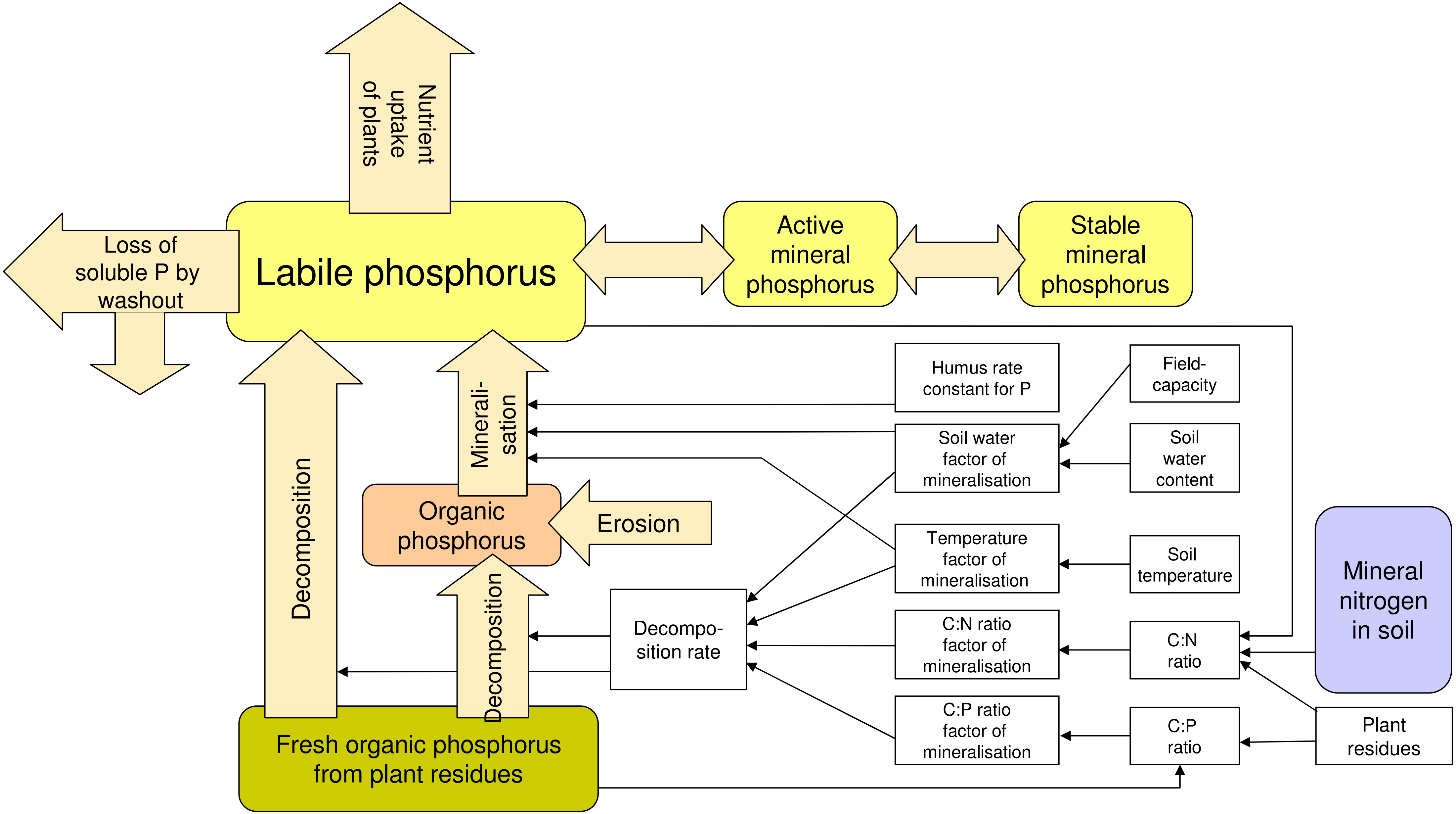
Crop yield module in SWIM



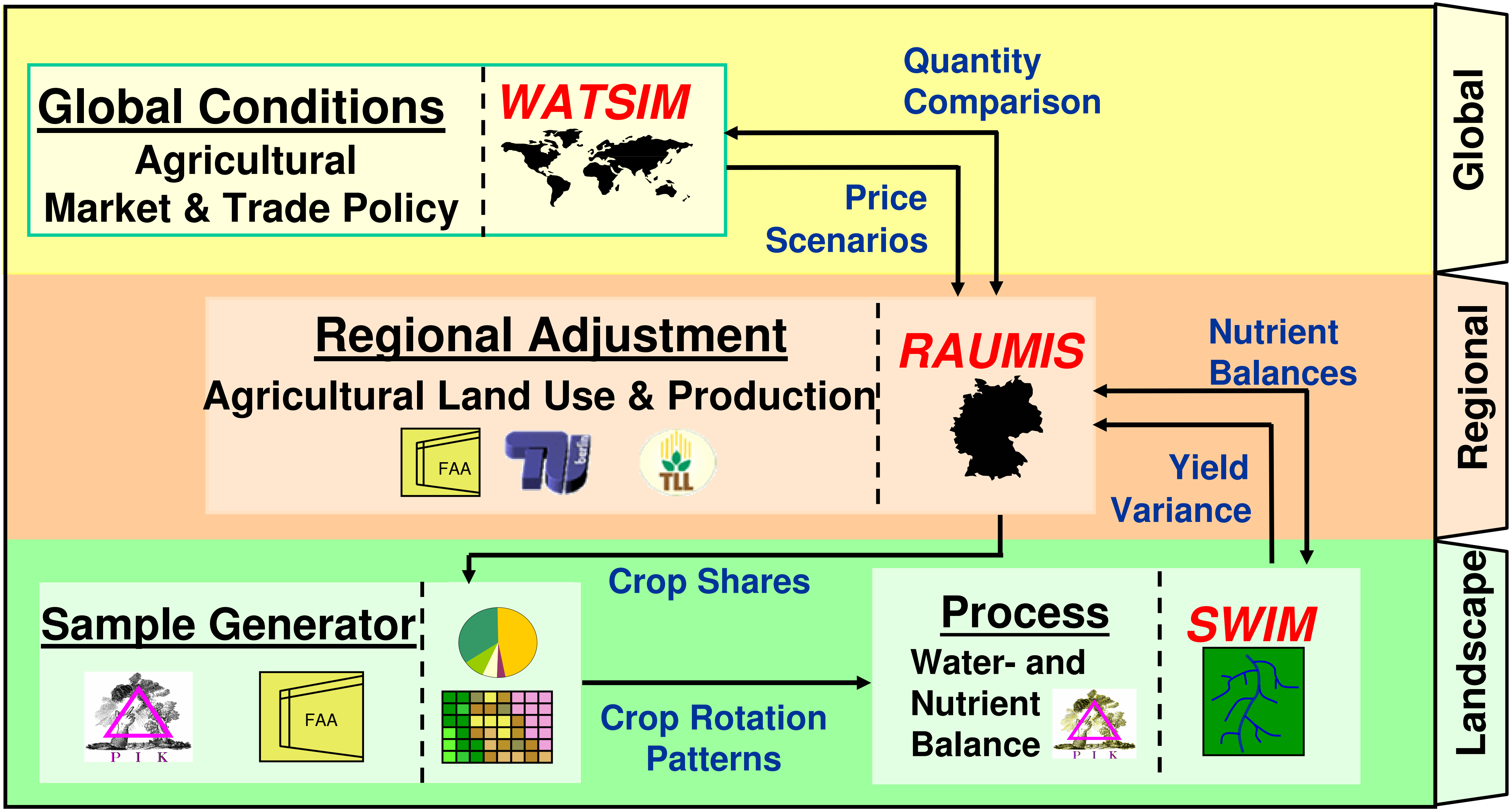
Nitrogen cycle in SWIM



Phosphorus cycle in SWIM



Model Coupling:
WATSIM ⇔ RAUMIS ⇔ SWIM



SWIM – application to riparian zones and wetlands

Example: Impact of wetlands and riparian zones on water and nitrogen dynamics at the basin scale (Nuthe basin, area of ca 2000 km^2)

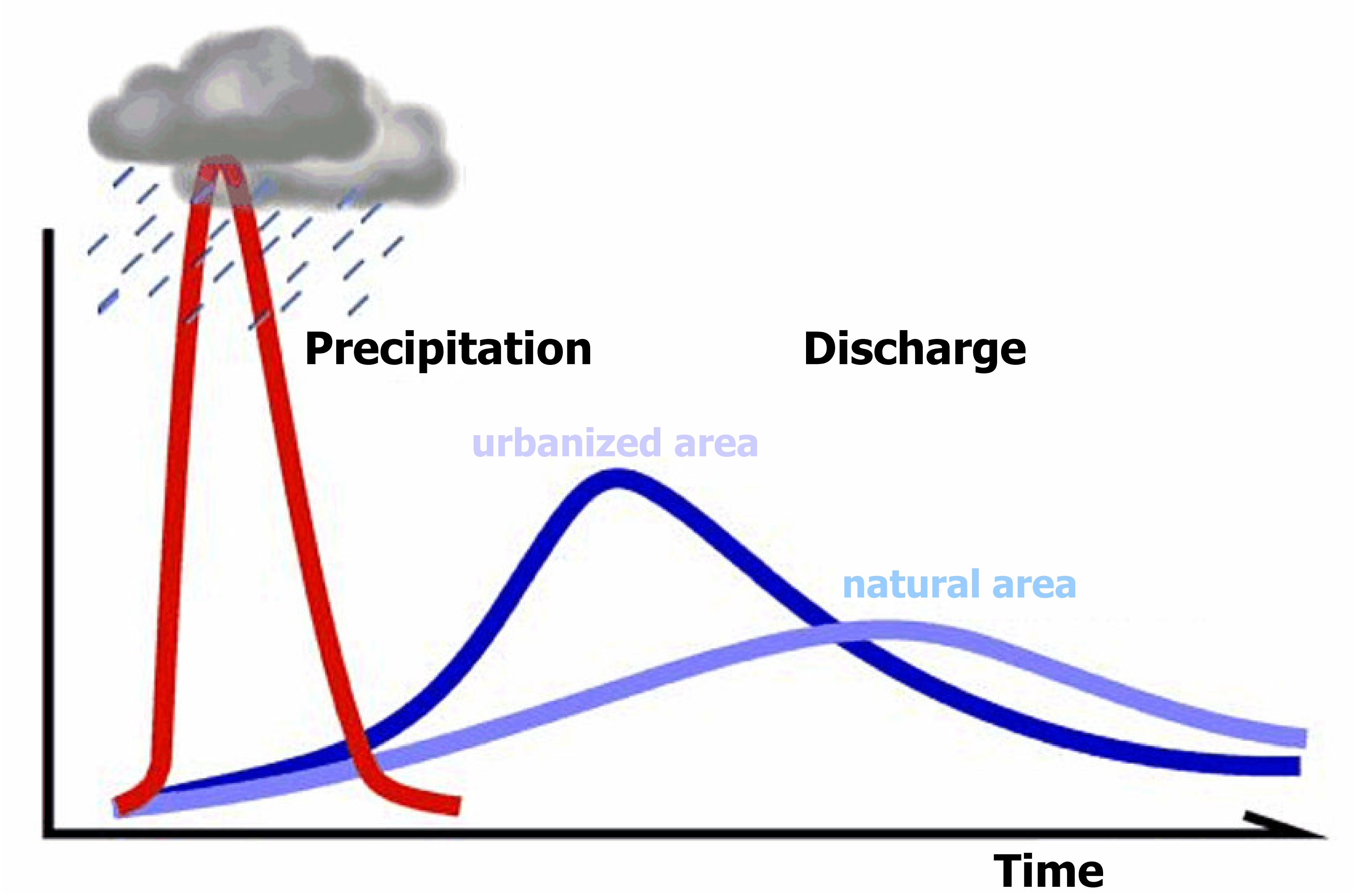
5 – Modelling in a changing world

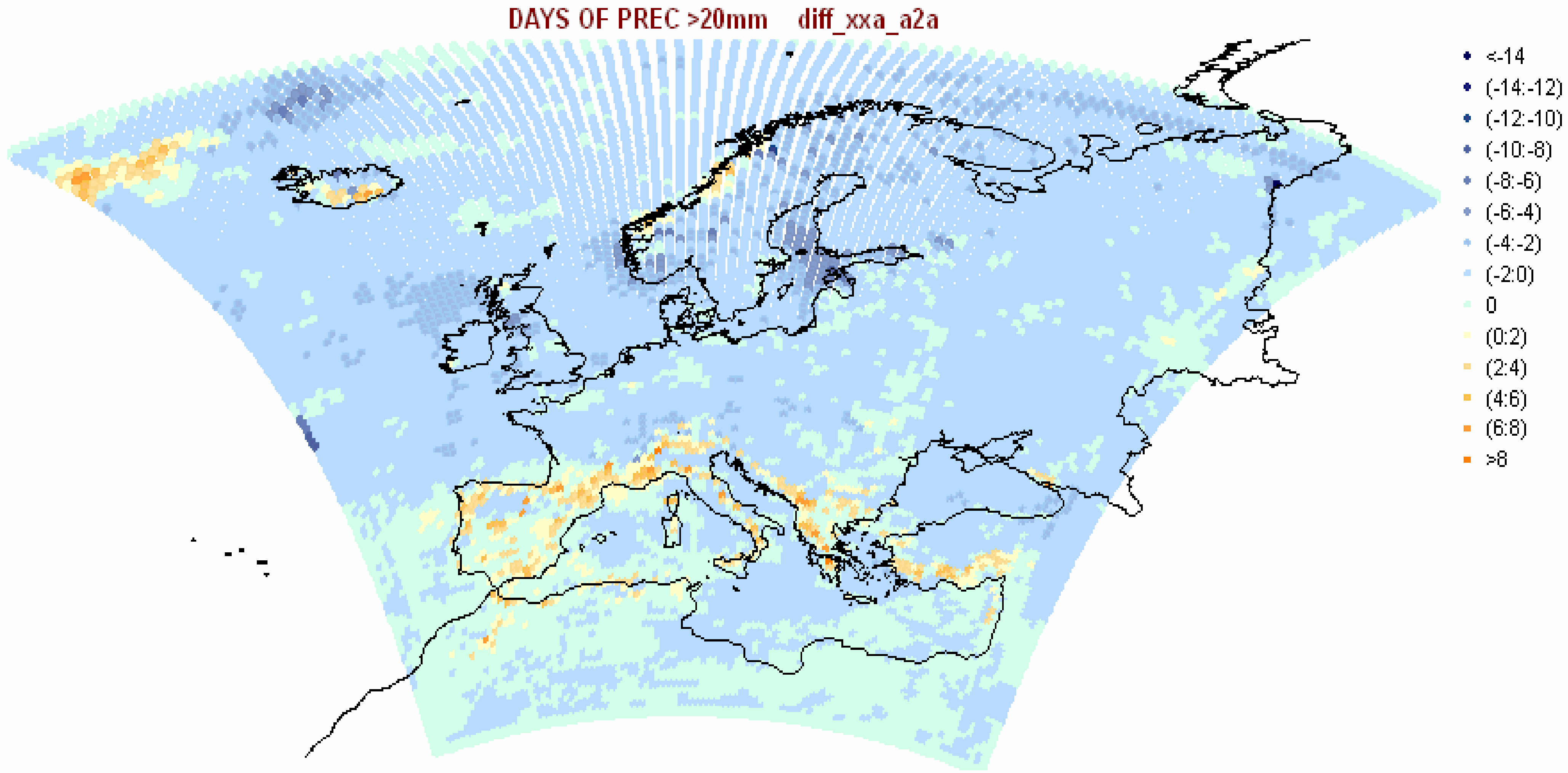
The convenient assumption of stationarity is clearly violated in the changing world. Many aspects of the system are subject to change, either in a natural or anthropogenic way (climate, land use, land cover).

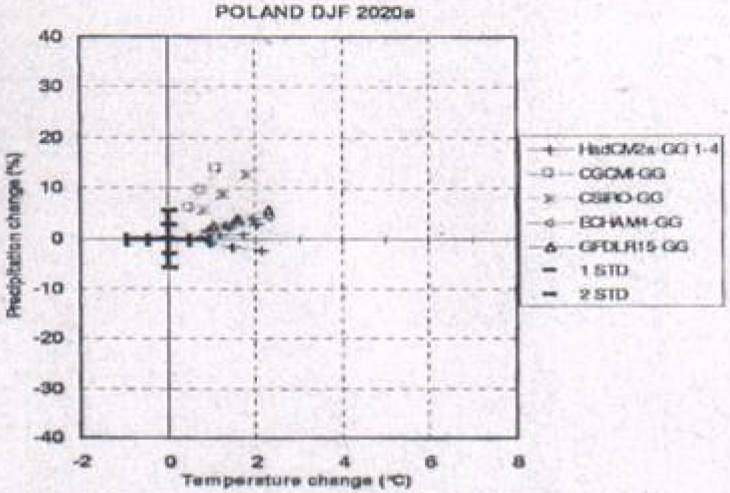
One can challenge the statement that the past is the key to the future.

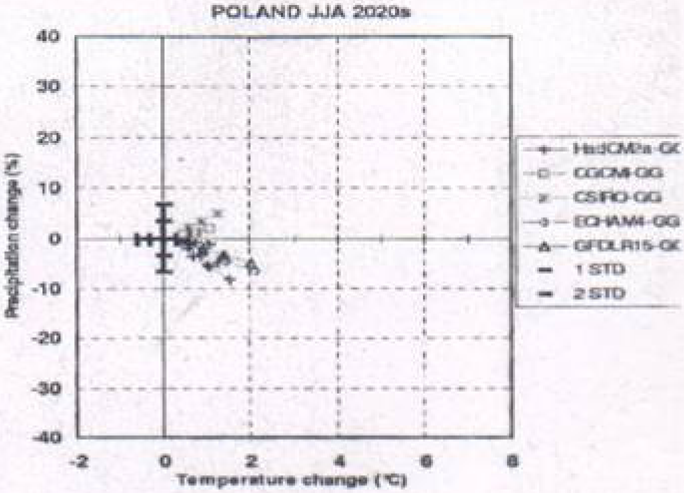
Mathematical models (calibrated for present conditions) are the principal way to study future situations. However, although purely empirical and black-box relationships continue to prove beneficial under certain circumstances, they may be subject to serious error when it becomes necessary to rely upon them under conditions not previously experienced.

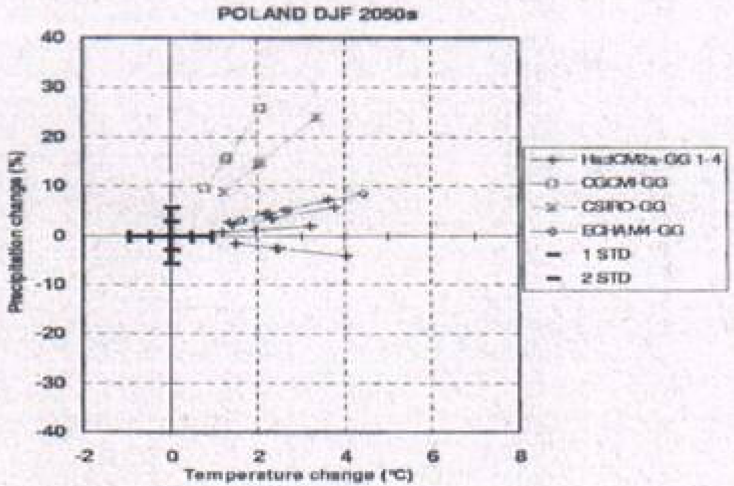
Hence, physically-based models, founded on theoretical concepts, are expected to be more trustworthy under such conditions, and experimentation with them holds greater promise for advancing the science.

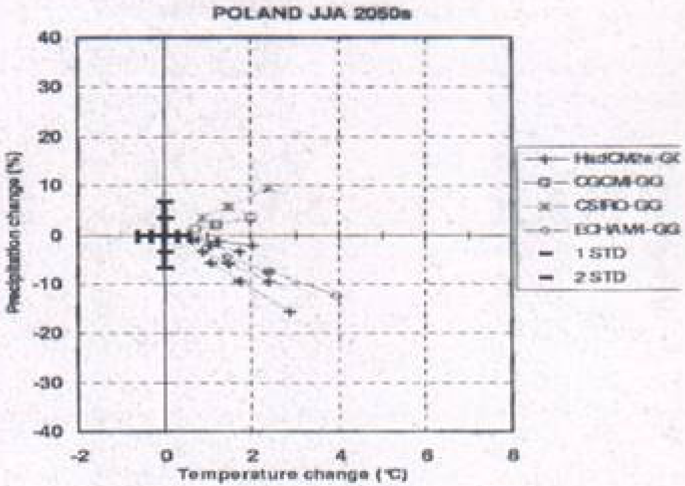

Precipitation
Discharge
urbanized area
natural area
Time

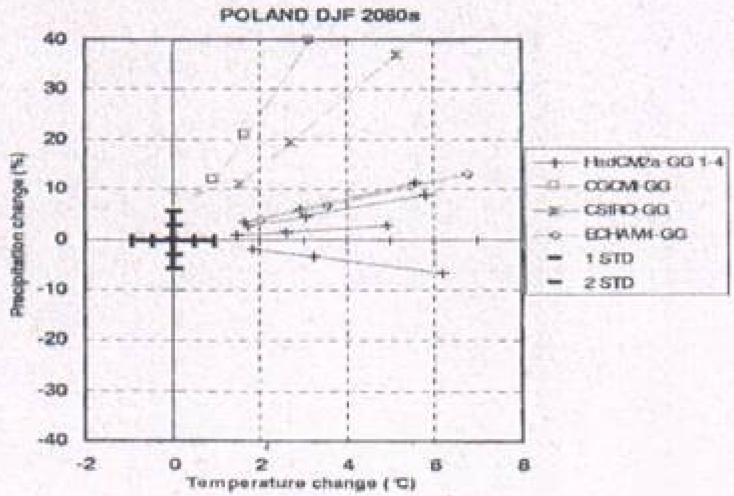

DAYS OF PREC >20mm diff_xxa_a2a
<-14
(-14:-12)
(-12:-10)
(-10:-8)
(-8:-6)
(-6:-4)
(-4:-2)
(-2:0)
0
(0:2)
(2:4)
(4:6)
(6:8)
>8

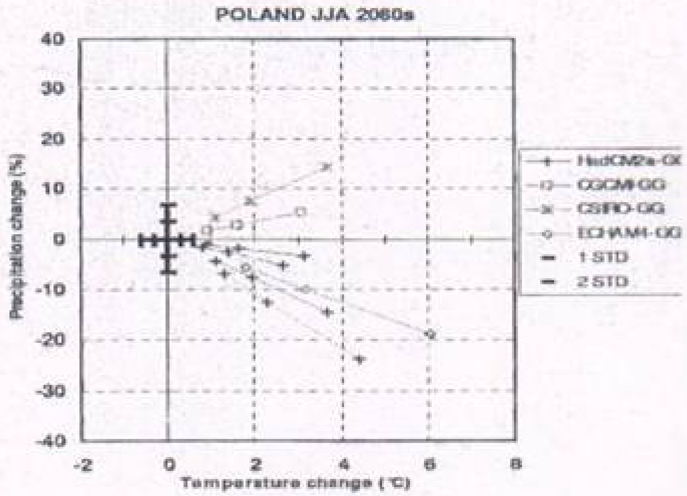

POLAND DJF 2020s
Precipitation change (%)
Temperature change (°C)
HadCM2a-GG 1-4
CGCMI-GG
CSIRO-GG
ECHAM4-GG
GFDLR15-GG
1 STD
2 STD


POLAND JJA 2020s
Precipitation change (%)
Temperature change (°C)
CGCMI-GG
CSIRO-GG
ECHAM4-GG
1 STD
2 STD


POLAND DJF 2050s
Precipitation change (%)
Temperature change (°C)
HadCM2a-GG 1-4
CGCMI-GG
CSIRO-GG
ECHAM4-GG
1 STD
2 STD


POLAND JJA 2050s
Precipitation change (%)
Temperature change (°C)
CGCMI-GG
CSIRO-GG
ECHAM4-GG
1 STD
2 STD


POLAND DJF 2080s
Precipitation change (%)
Temperature change (°C)
HadCM2a-GG 1-4
CGCMI-GG
CSIRO-GG
ECHAM4-GG
1 STD
2 STD


POLAND JJA 2080s
Precipitation change (%)
Temperature change (°C)
CGCMI-GG
CSIRO-GG
ECHAM4-GG
1 STD
2 STD

6 – Final remarks

There are many models on the market, products of research institutions and commercial software companies. It is often difficult to ascertain the relative advantages and disadvantages of models proposed for operational use.

Among factors and criteria involved in the selection of a model are the following:

- The general modelling objective: e.g. hydrological forecasting, assessing human influences on the natural hydrological regime, or climate change impact assessment;
- The type of system to be modelled: small watershed, aquifer, river reach, reservoir, or large catchment;
- The hydrological element to be modelled: floods, daily average discharges, monthly average discharges, groundwater levels, water quality, etc.;
- The type of model and the description of the most relevant hydrological processes, for instance:
  - If minimum streamflow is being investigated in a catchment then the model should adequately represent groundwater;
  - It is very important that a forecasting model should contain an updating component;

- The climatic and physiographical characteristics of the watershed;
- Data availability (their type, length, and quality) vs data requirements for model calibration and operation
- The possible need for transposing model parameters from smaller catchments to larger catchments;
-The criterion of the availability and size of computers for both development and operation of the model has largely lost its importance with the present generation of inexpensive PCs;
- Model availability and user-friendliness, know-how;
- Model simplicity (hydrological complexity, ease of application).

Model intercomparison. Beauty contest? Criterion – how do models do their job?

Models with balanced components are advocated – the weakest link drives the quality of the whole, hence making the best component even better may not influence the overall quality, while elimination of a bottleneck may lead to a visible improvement.

Vit Klemes: Mathematistry – abuse of mathematics, development of a (complicated) model as an aim per se (for fun?)

When selecting models, and using them, remember about:

- range of applicability
- assumptions taken in model development